Scheduling Problems with Equipment Health Index

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1 Introduction

Most of the papers in the literature on scheduling consider that a machine is fully available or not at all. However, even if the machine is available, it may not be in a perfect state and the process quality may not be guaranteed. The objective is to take the degradation of the machine into account when scheduling jobs and maintenance operations. Most machines are now equipped with sensors to evaluate their current status. This is for instance true in the semiconductor industry, where studies show the impact of the health of the machine on the process quality. This work considers a single machine with a health index which decreases when a job is processed on the machine. If the health index is below a certain level, then jobs no longer can be sequenced and a maintenance operation is required. This leads to an "as good as new" state, restoring the health index at its maximum level. Maintenance operations have a fixed duration but their start times are flexible and thus can be optimized. Jobs are grouped in families which correspond to different types of products. All jobs in the same family have the same processing time and the same health index requirement. A job can only be started on the machine if the current health index of the machine is larger than the health index requirement plus the processing time of the job. The objective is to minimize the flow time, i.e. the sum of completion times.

Two cases were studied : The daily case, where a single maintenance operation is necessary in the schedule to restore the health of the machine ; and the weekly case, where two maintenance operations might be necessary.

Figure (1) shows an illustrative example of schedule for the daily case. The jobs of the same family have the same number and the same color. The grey boxes represent the maintenance operation. The numbers above the jobs are the health indices of the machine at the beginning and at the end of each job. The triangles downward are the health index requirements of the families. After the maintenance operation, the health is at its maximum.

2 Models

To minimize the sum of completion times, two Mixed Integer Linear Programming models are presented for each case. The first type of models uses "classical" positional variables in the

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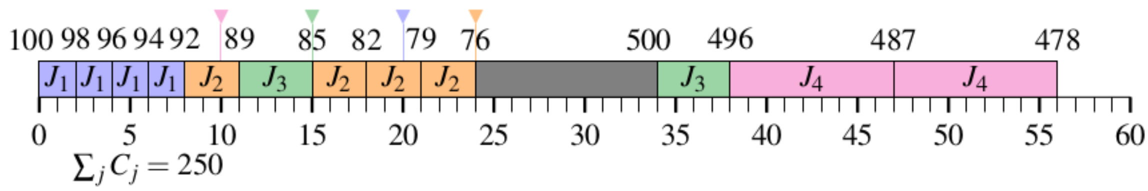

FIG. 1 – Example of a schedule with 4 families for the daily case.

literature, while the second type of models improves the first one by using the notion of master sequence. Valid inequalities are also proposed.

For the first type of models, two MILP models named "MILP-base", for the daily and weekly cases, use positional variables, i.e. binary variables that define the positions of jobs in the sequence. Each job is assigned to at most one position, and a position can be occupied by at most one job. We define n positions before the first maintenance operation, n positions after the first maintenance operations and n positions after the second maintenance operation for the weekly case with n the number of jobs. One type of valid inequality is developed to strengthen the formulations, named "Cuts based on dominant positions".

The second type of models is based on the notion of master sequence. Two theorems are provided. They specify how jobs are ordered in an optimal solution according to their processing times and their health index requirements. With these two theorems, an algorithm is proposed to construct the master sequence, which is a sequence of jobs from which at least one optimal sequence can be generated. Properties to reduce the length of the master sequence are introduced.

Two Mixed Integer Linear Programming models for the daily and weekly cases are derived from the master sequence, named "MILP-MS". The decision variables depend on the master sequence. Here, the positions selected in the sequence imply the schedule. Four types of valid inequalities are introduced to strengthen the models. The first one is about the same as the one used for the "MILP-base" model, named "Cuts based on dominant positions".

3 Results

The numerical results showed that the MILP models based on the master sequence strongly dominate the first MILP models, and that reducing the length of the master sequence is very effective, in particular when there are many jobs. "Cuts based on dominant positions" significantly decrease the computational times for all models. For MILP-MS, the three other types of valid inequalities reduce the computational times. Then a comparison was done for the weekly case. For the same number of families and jobs, the computational times are larger than for the daily case. MILP-MS with all valid inequalities and improvements is still more efficient than MILP-base with its valid inequalities.

4 Other results

The complexity of the daily case was studied during the internship but remains open. We showed that the two-family case is solvable in polynomial time and gave an algorithm. We gave a NP-complete proof for the weekly case. We developed a heuristic for the daily case based on two well-know algorithms of the literature, Moore's algorithm and Smith's algorithm. The numerical results showed that the heuristic is efficient when the maintenance duration is large enough. A study was made on the weighted case, where the objective is to minimize the sum of the weighted completion times for MILP-MS. It has been shown through counterexamples that the notion of master sequence cannot be adapted, and thus also the associated MILP models.