On the Maximum Flow Blocker Problem

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1 Introduction

We are interested in the relation between blocker and interdiction applied to the Maximum Flow. Let $G = (V, A)$ be a graph where V is the set of nodes and A is the set of arcs, containing a source $s \in V$ and a destination $t \in V$. Each arc $a \in A$ has a capacity c_a and a cost r_a called *interdiction cost*.
The **Maximum Flow Interdiction Problem** consists in finding a set of arcs to delete from the network in order to minimize the maximum flow between s and t. These arcs, called *interdicted arcs* are restricted by an *interdiction budget*. In other words, the total interdiction cost generated by the interdicted arcs should be less than or equal to the interdiction budget, denoted by R. We note this problem MFIP(G, c, r, R).
The **Maximum Flow Blocker Problem** consists in finding a set of arcs with a minimum total interdiction cost and such that maximum flow remaining between s and t does not exceed a value called, *target flow value* and denoted by F. We note this problem MFBP(G, c, r, F).
Unlike the Maximum Flow Blocker Problem, the interdiction variant has been largely studied in the literature. We refer the interested reader to [1] and [2] for an overview of works that have been done around the Maximum Flow Interdiction Problem. Precisely, in [1], the author proposes a compact integer formulation to solve MFIP(G, c, r, R).

2 Solution approach

We denote by $\delta(U)$ an $s - t$ cut of a graph $G = (V, A)$ corresponding to the set of arcs in G with the head in U, the tail in $V \setminus U$ and such that $s \in U$ and $t \in V \setminus U$. The following theorem states a relation between the solution of a Maximum Flow Blocker Problem and the cuts of a graph.

Theorem 1. *Given a graph $G = (V, A)$, any optimal solution for the blocker problem MFBP(G, c, r, F) is contained in an $s - t$ cut of G.*

Figure 1 represents solution of a Maximum Flow Blocker Problem with a target flow value $F = 23$. On each arc is reported the interdiction cost (above), the flow on the arc (first value below) and the capacity (second value below). The interdicted arcs are represented by dashed blue lines. The red thicker arcs define a minimum $s-t$ cut in the remaining graph. These arcs with the interdicted arcs constitutes an $s-t$ cut in the graph $G=(V,A)$. Remark that the maximum flow remaining in the graph is equal to 18, which is less than the target flow value.

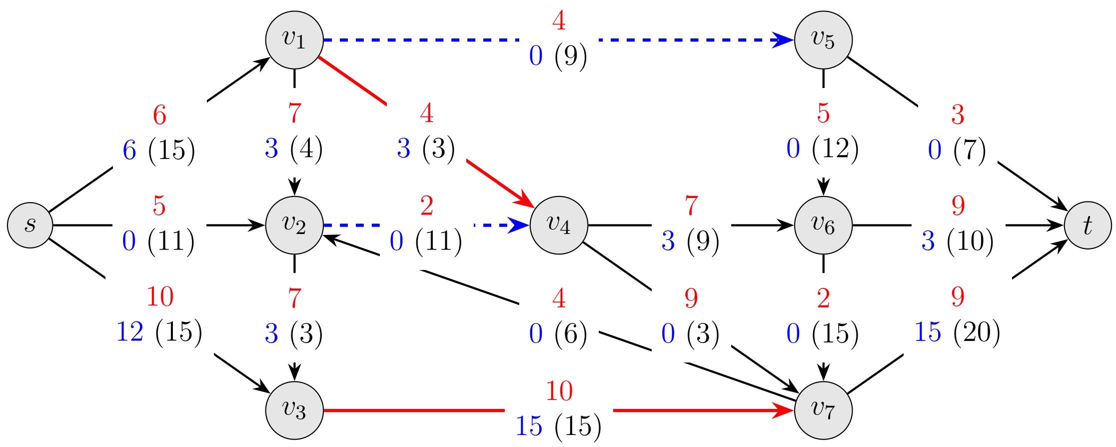

FIG. 1 – Example of a Maximum Flow Blocker Problem with a target flow value $F=23$.

To the best of our knowledge, the Maximum Flow Blocker Problem has not been studied in the literature. In our conducting works, we show that there is an equivalence relation between the Maximum Flow Blocker Problem and the Maximum Flow Interdiction problem, given by the following theorem.

Theorem 2. *A^I is an optimal solution for the interdiction problem MFIP(G,c^I,r^I,R) if and only if the $s-t$ cut containing the arcs A^I is an optimal solution for the blocker problem MFBP$(G,c=r^I,r=c^I,F=R)$.*

Références

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