Guaranteed-performance of robust algorithms for solving combinatorial optimization problems with imprecise and changing data

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Keywords : *Scheduling problems, Robust optimization, Approximation algorithms*.

1 Introduction

Scheduling problems arise in many real-world applications, e.g., in the fields of economy, industry, or transport logistics. For many such problems, fast algorithms have been developed with the condition that all input parameters are known in advance. However, in practical applications, problems become more complex when considering uncertainty in the input data, which may result from the limited access to information, estimated data or simple measuring errors. One successful approach to tackle uncertainty in problem data is robust optimization, which deals with the deterministic set-based representation of the uncertainty without any probabilistic description, in order to produce decisions that will have a reasonable objective value under any possible realization of parameters (called scenario). In this paper, we apply the robustness approach to examine the fundamental single machine sequencing problem defined in section 2. In section 3, we describe our structuring of data uncertainty. In section 4, we discuss the choice of the robustness criterion. We present literature review in section 5 and our results in section 6.

2 Problem description

The deterministic version of the considered scheduling problem is denoted by $1|r_j|L_{max}$ in Graham's notation. Given a set $J = \{1, 2, \ldots, n\}$ of n jobs to be processed on a single machine. Each job $j \in J$ is described by the processing time p_j, the release date r_j and the delivery time q_j. The completion time of the job j is denoted by C_j and its lateness is calculated as follows $L_j = C_j + q_j$. The objective is to find a feasible sequence for which the maximal lateness is minimal.

3 Uncertainty set

The concept of scenario is a common and natural way to structure uncertainty. Indeed, several methods of defining the scenario set have been proposed in the literature. Among the simplest and most popular ones are the discrete and interval uncertainty. We are interested especially in the interval uncertainty representation where the value of each parameter is known to fall within a closed interval, i.e. p_j may fall within the range bounded by p_j^{min} and p_j^{max}. The scenario set is the Cartesian product of all these intervals.

4 Robustness Criteria

Let Γ denote the set of all possible scenarios and Π the set of all the feasible sequences in the problem. The maximal lateness in sequence $\pi \in \Pi$ under scenario $s \in \Gamma$ is denoted as $L(\pi, s)$ and the value of an optimal schedule under a fixed scenario s, as $L^*(s)$. Different criteria can be used to select among robust decisions:

- **Min-Max criterion:** aims at constructing sequences having the best possible performance in the worst case scenario, i.e.,

$$\underset{\pi\in\Pi}{Min}\ \underset{s\in\Gamma}{Max}\ L(\pi, s) \qquad (1)$$

This research benefited from the support of the FMJH Program PGMO and from the support to this program from EDF-THALES-ORANGE. This research benefited from the support of ANR Lorraine Artificial Intelligence (ANR-LOR-AI Project).

- **Min-Max Regret criterion:** less conservative, aims at obtaining the best worst case deviation from optimality, over all possible scenarios, i.e.,

$$\underset{\pi\in\Pi}{Min}\ \underset{s\in\Gamma}{Max}\left(L(\pi,s)-L^{*}(s)\right) \qquad (2)$$

- **Min-Max Relative Regret criterion:** aims at obtaining the worst case percentage deviation from optimality, over all possible scenarios, i.e.,

$$\underset{\pi\in\Pi}{Min}\ \underset{s\in\Gamma}{Max}\ \frac{L(\pi,s)}{L^{*}(s)} \qquad (3)$$

5 Literature review

The majority of researchers focus on single machine scheduling problems with the Min-Max regret criterion according to different objectives. For instance, Kasperski, in [1], developed a polynomial time algorithm for the Min-Max Regret $1 \mid prec \mid L_{max}$ problem, where uncertain due dates and processing times are represented as interval data, and in [2], shows that the Min-Max Regret $1 \mid prec \mid \sum C_j$ problem, where processing times are represented as interval data, is approximable within 2, if the deterministic problem is polynomially solvable.

6 Our results

Polynomial algorithms are constructed for problems presented in Table 1. Indeed, we proved some dominance rules which helped us first to determine the worst case scenario of any solution and then to choose the solution that minimizing the maximum (relative) regret.

Criterion	Basic problem	Uncertain parameters
Min-max Regret	$1\|r_j,p_j=p\|L_{max}$	$r_j^s \in [r_j^{min},r_j^{max}]$ *and* $q_j^s \in [q_j^{min},q_j^{max}]$
Min-max Relative Regret	$1\|r_j,p_j=p\|L_{max}$	$r_j^s \in [r_j^{min},r_j^{max}]$ *and* $q_j^s \in [q_j^{min},q_j^{max}]$
Min-max Relative Regret	$1\|prec\|L_{max}$	$q_j^s \in [q_j^{min},q_j^{max}]$

Table 1 : Solved problems

7 Conclusion and perspective

In this paper, we examine the fundamental single machine sequencing problem $1|r_j|L_{max}$. Interval uncertainty can be related to processing time, release date or delivery time. So that, depending on the choice of the robust criterion and according to several constraints, we can define a lot of variant of this problem. Up to now, we have solved some cases and we are working on others. Especially, we intend to treat the Min-Max Relative Regret criterion which allows us to tackle any robust optimization problem despite the NP-hardness of its basic problem, contrary to the Min-Max Regret criterion which becomes not approximable, unless P=NP, when the basic problem is NP-hard.

References

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[2] A. Kasperski and P. Zielinski. A 2-approximation algorithm for interval data minmax regret sequencing problems with the total flow time criterion. *Operations research letters, 2008*.