Mixed-Integer Programming for the ROADEF/EURO 2020 challenge

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1 Introduction

The ROADEF 2020 challenge presents a maintenance scheduling problem from the French electricity grid company RTE. The modeling of uncertainty makes the problem highly non-convex and apparently out of the reach of mathematical solvers. We present our approach for the challenge problem. It is based on a new family of cutting planes, coupled with a constraint generation approach. We present mathematical proofs and separation algorithms for the cutting planes. We then study the practical impact of our additions on the challenge instances, showing that our approach significantly reduces the optimality gap obtained by the solver.

2 Constraint generation

In order to speed-up the solution process, we devise a constraint generation approach, similar to an integer Bender's decomposition [1]. With the quantile function as a subproblem, we obtain a simple formula to derive the most violated constraint.

3 Cutting planes

We attempt to strengthen the MILP formulation with the addition of cutting planes. We write $Q_k(x)$ the k^{th} largest element of the vector $x \in \mathcal{R}^n$.

We analyze the convex envelope of the quantile function and derive a family of cutting planes and an efficient separation algorithm. However, this first family of cutting planes is weak in practice.

Cutting plane 3.1 *Suppose $l_i \leq x_i \leq u_i$. We write $L = Q_k(l)$. Then, given a subset P of $[1 \mathinner{..} n]$ with at least k elements and $U > L$,*

$$Q_k(x) \geq L + \frac{U - L}{|P| - k + 1} \left(\sum_{i \in P} \frac{x_i - L}{\max(U, u_i) - L} - k + 1 \right)$$

To obtain useful cutting planes, we take into account the fact that the quantile function is applied on the result a linear function. This yields a much more efficient cutting planes family. The separation problem of this new family is NP-hard, and we developed heuristics to apply these cutting planes.

Cutting plane 3.2 *Suppose $l_i \leq x_i \leq u_i$. Then, given a subset P of $[1 .. n]$ with at least k elements and $\beta \in [0,1]^n$: :*

$$\begin{aligned} Q_k(Ax) \geq & \sum_{j=1}^{m} \min_{i \in P}(a_{ij})(1-\beta_j)(x_j - l_j) \\ & + \sum_{j=1}^{m} \max_{i \in P}(a_{ij})\beta_j(x_j - u_j) \\ & + \min_{i \in P} \sum_{j=1}^{m} a_{ij} \left((1-\beta_j)l_j + \beta_j u_j\right) \end{aligned}$$

4 Conclusions

The constraint generation approach allows us to apply a MILP solver to large instances of the challenge. Coupled with the cutting planes, we obtain much smaller optimality gaps than the natural formulation of the problem, while reaching solutions competitive with dedicated local search algorithms on many instances.

Références

[1] Ashkan Fakhri, Mehdi Ghatee, Antonios Fragkogios, and Georgios K. D. Saharidis. Benders decomposition with integer subproblem. *Expert Syst. Appl.*, 89 :20–30, 2017.