Picker routing problem in Mixed-shelves warehouses with multiple cross aisles

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1 Introduction and problem statement

The picker routing problem in mixed shelves warehouses is defined in a warehouse of business-to-consumer (B2C) segment characterized by:

- **a manual picker to part system** in which an order picker starts from a central **depot**, walks through **picking** and **cross-aisles** to pick items (stock keeping units) stored in **shelves** (storage locations), and comes back to the depot. During a picking tour, the picker processes a single order (pick by order) or a compilation of orders (pick by batch). In both cases, he/she is guided by a "picklist" in which the requested stock-keeping units (SKUs) are listed.
- **a mixed-shelves storage paradigm** where unit loads of an SKU are broken down into single items, that are randomly spread throughout the warehouse. Hence, each SKU is available in multiple storage locations and each storage location can handle multiple SKUs.

Few articles in the literature study the picker routing problem in mixed-shelves warehouses. To the best of our knowleadge, [1] are the only ones that tackle the problem using an exact approach. They develop a warehouse-based ILP that exploits characteristics of an optimal solution of the classical picker routing problem. However, their method is specially designed for warehouses with one bloc (2 cross aisles) and is not appliable to warehouses with multiple blocs. The main contribution of our work is to propose an exact method that handles warehouses with multiple blocs.

Formally, We consider a picklist $\mathcal{I} = \{1, \cdots, s, \cdots, S\}$ in which the SKUs to pick in a single picking tour are itemized. Each SKU $s \in \mathcal{I}$ is requested in r_s units, and is available in a set of storage locations $\mathcal{L}_s$. Let $\mathcal{L}$ be the set of all storage locations associated with the SKUs of $\mathcal{I}$ (*i.e.* $\mathcal{L} = \bigcup_{s\in\mathcal{I}} \mathcal{L}_s$). In each storage location $l \in \mathcal{L}$, a set $\mathcal{S}_l$ of SKUs is available in limited quantities. The number of SKUs $s \in \mathcal{S}_l$ is denoted q_l^s. Finally, let define $\mathcal{G} = (\mathcal{P}, \mathcal{E})$ a complete and undirected graph modeling the warehouse. $\mathcal{P} = \{0\} \cup \mathcal{L}$ is the set of positions in the warehouse with 0 representing the depot. We associate a travel distance $d_{p,p'}$ for each edge $(p, p') \in \mathcal{E}$.

The objective of the problem is to select the positions from where to pick the SKUs and design the tour that visits the selected positions. Note that due to the limited quantities of SKUs stored in each storage location, the picker could visit multiple storage locations to pick the units of each requested

SKU. Let $\pi = (0, 1, \cdots, p, \cdots, P, 0)$ be a solution representation of our problem. π is feasible if the quantity of SKUs retrieved from the selected storage locations covers all the requested SKUs in the picking list $\mathcal{I}$. Let $f(\pi)$ the function that returns the total travel distance of π. We seek for π^{opt} the solution that minimizes the total distance traveled:

$$\pi^{opt} = argmin_{\pi} \left(f(\pi) = d_{0,1} + \sum_{p=1}^{P-1} d_{p,p+1} + d_{P,0} \right) \tag{1}$$

2 Logic-Based Benders Decomposition

We propose a Logic-based decomposition procedure to solve the problem. LBBD is a generalization of the classical Benders decomposition procedure where the sub-problem is not continuous, and hence can not be solved using the linear programming duality. Instead, LBBD derives Benders cuts by solving the more general inference dual ([2]). The solution of the inference dual provides a bounding function on the cost of the master problem that is tight for its current solution. Note that no systematic procedure is available to derive such cuts as in the case of classical Benders. Tailored cuts must be developed according to the studied problem.

At each iteration of our LBBD procedure, we first select in the master problem a set of locations to visit such as the demand r_s of each item $i \in \mathcal{I}$ is satisfied. Let $\bar{\mathcal{H}}$ be the selected locations. Next, we call an oracle that computes the shortest elementary cycle that visits all the locations in $\bar{\mathcal{H}}$. Note that for each $\bar{\mathcal{H}} \subseteq \mathcal{P}$, the sub-problem is feasible. Hence, only optimality cuts are generated from the sub-problem. Let $\bar{\theta}$ be the total distance of the optimal tour returned by the oracle. we then update the lower and upper bounds and add the constraint (2) to the master problem ($x_p,\ p \in \mathcal{I}$) is a binary variable that equals 1 if we select the location p, 0 otherwise). We repeat these operations until the lower bound equals the upper bound. Finally, note that some acceleration techniques are proposed and implemented to speed up the procedure (they are called: lower bounding function, valid inequalities, preprocessing).

$$\theta \geq \bar{\theta} - \sum_{p \in \bar{\mathcal{H}}} \left[(1 - x_p)(2 \cdot \max_{p' \in \bar{\mathcal{H}}, p' \neq p} \{d_{p,p'}\}) \right] \tag{2}$$

3 Computational experiments

To test the performance of our procedure (LBDP-MS), we generated a set of instances that takes into account several warehouse parameters (number of aisles, number of cross aisles, size of pick list, the degree of duplication of the SKUs in the warehouse). We compared LBDP-MS with a method called ILP-MS that is an adaptation of a powerful ILP formulation initially designed for the classic picker routing problem to the mixed-shelves warehouses. Preliminary experiments show that LBDP-MS returns promising results and outperforms ILP-MS.

References

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