An extended MILP formulation for the design of modular multi-model reconfigurable manufacturing lines

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1 Context

In this study, we consider the multi-model assembly line balancing problem (MuMALBP) with modular workstations. The MuMALBP arises when different types of product have to be performed on a single paced line. However, in order to be produced, each type of product requires a particular line configuration, which is often subject to different restrictions such as cycle time and task precedence constraints. These latter are usually represented by a directed acyclic graph. Moving from the manufacturing of one product to another implies a reconfiguration of the line, which consists in relocating some tasks between the existing workstations in order to fulfill the requirements of the new product type. To do this, the MuMALBP is usually converted to the well-known simple assembly line balancing problem by combining the precedence graph of each product into a single one. However, this approach is not efficient when the products have significant differences, which is the case in the current market environment. In this circumstance, a solution is to take into account all precedence graphs without modification and design a reconfigurable line having a specific configuration for each product so as probable task relocations are optimized.

According to [1], modularity is one of the main characteristics of reconfigurable manufacturing systems. Modularity consists in subdividing operational tasks into interdependent units, or modules, so that they can be easily changed, upgraded or replaced without impacting the rest of the system. Thus, modularity provides the system with a flexibility that allows it to easily cope with market fluctuations and changes. Despite all the advantages of modularity, its integration into the design of manufacturing lines remains a very complex problem with only few studies in the literature. The present abstract aims to fill this gap. More precisely, we consider the MuMALBP with a fixed number of workstations equipped with modules, where each of which is activated successively. Each module may perform a limited number of tasks sequentially. The working time of one module is computed as the sum of the processing time of the tasks assigned to it. As a consequence, the workload of any workstation is calculated as the sum of the working time of all its modules. Thus, for the considered line, moving from one configuration to another requires adding, removing or moving modules (and not tasks) between the available workstations.

2 Studied problem

The above description rises an important optimization problem, which is to find an admissible line configuration for each product type while minimizing the total number of used modules.

In order to tackle such a problem, we at first developed a compact mixed-integer linear programming (MILP) formulation. This formulation was not efficient as it provided poor quality results, in terms of computational CPU time and optimality GAP, even for small size instances. As a consequence, we then designed a more efficient extended MILP formulation, which is based on the fact that the set of all possible modules is already known. This formulation (1)-(7) is presented below.

$$\min \sum_{m\in M} y_m \tag{1}$$

$$\sum_{k\in W}\sum_{m\in M} \alpha_{im}\cdot\lambda_m^{(k,p)} = 1, \quad \forall i\in V, \quad \forall p\in P \tag{2}$$

$$\sum_{m\in M} c_m^{(p)}\cdot\lambda_m^{(k,p)} \le C^{(p)}, \quad \forall k\in W, \quad \forall p\in P \tag{3}$$

$$\lambda_m^{(k,p)} \le y_m, \quad \forall m\in M, \quad \forall k\in W, \quad \forall p\in P \tag{4}$$

$$\sum_{k\in W} \lambda_m^{(k,p)} \ge y_m, \quad \forall m\in M, \quad \forall p\in P \tag{5}$$

$$\sum_{q=k}^{|W|}\sum_{m\in M} \alpha_{im}\cdot\lambda_m^{(q,p)} \le \sum_{q=k}^{|W|}\sum_{m\in M} \alpha_{jm}\cdot\lambda_m^{(q,p)}, \quad \forall(i,j)\in A^{(p)},\ \forall k\in W,\ \forall p\in P \tag{6}$$

$$\lambda_m^{(k,p)} = 0,\ \forall m\notin M^{(k,p)},\ \forall k\in W,\ \forall p\in P \tag{7}$$

$$y_m\in\{0,1\}, \quad \forall m\in M$$

$$\lambda_m^{(k,p)}\in\{0,1\}, \quad \forall m\in M, \quad \forall k\in W, \quad \forall p\in P$$

The objective function (1) minimizes the total number of modules, where M is a set of all generated modules and y_m is a decision variable that is equal to 1 if module $m \in M$, 0 otherwise. Constraints (2) ensure that each task $i \in V$ is assigned only once in a configuration corresponding to product $p \in P$. Here, $\lambda_m^{(k,p)}$ is a decision variable, which is equal to 1 if module $m \in M$ is assigned to workstation $k \in W$ in the configuration corresponding product $p \in P$. Cycle time and precedence constraints are expressed using inequalities (3) and (6), respectively. Constraints (4) and (5) consider that a module $m \in M$ is used if it is assigned to at least one configuration $p \in P$. As for constraints (7), they are valid inequalities that indicate for each module its assignment interval within configuration $p \in P$.

The above-described extended MILP formulation was able to solve to optimality all small size instances in less than 1 second on average. As for medium size instances, the formulation (1)-(7) provided very promising results. The detailed comparison between the compact and extended MILP formulations as well as an original algorithm for intelligent generating and filtering all possible modules will be presented and discussed during the conference presentation.

References

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