Multi Item Capacitated Lot Sizing with Stochastic Demand Timing

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1 Introduction

Since the late 1950s, lot sizing problems have been extensively studied by researchers and practitioners, leading to a significant body of literature. While deterministic lot sizing problems constitute the majority of these studies, dealing with uncertainties has been a challenging task. As Brahimi et al. [2] summarize in their review, the demand quantity is the parameter that is mostly considered as uncertain in the stochastic lot sizing literature. Even though they are quite common in practical settings, stochastic lead times have been rarely studied, although [3, 4, 5] can be given as examples.

A significant limitation in stochastic lot sizing is that stochastic demands are assumed to be independent random variables, i.e the correlation between stochastic demands in different periods, which usually exists in real-life settings, is ignored. To build this correlation, Akartunalı and Dauzère-Pérès [1] consider that demand quantities are deterministic but their timing is stochastic. In [1], not only novel way of modeling uncertainty on demand is presented but also dynamic programs are proposed for several cases to solve the single-item dynamic lot sizing problem.

In this work, we extend the work of [1] to multiple items and propose appropriate solution approaches.

2 Problem Statement

In this work, the multi-item capacitated lot sizing problem with stochastic demand timing is addressed. Accordingly, the production horizon is divided into T periods and multiple items (N) are produced. Each period has a given capacity. The quantity and the period of the deterministic demands of item k (D_{kt}) are known in advance. No backlog or lost sale is allowed for deterministic demands. The quantity of the stochastic demands (d_i) of item $k(i)$ is also known. However, the timing of demand d_i is stochastic within a given time interval $[l^i, u^i]$ $\subseteq$[1,T], where it is certain that d_i will fully occur at once, with a probability of $p_t^i \geq 0$ for each period $t \in [l^i, u^i]$ and such that $\sum_{t=l^i}^{u^i}(p_t^i) = 1$ and $p_t^i = 0$ for $t \leq l^i - 1$ and $t > u^i + 1$. Backlog is allowed for the stochastic demand d_i until period u^i, and d^i can be satisfied from the inventory carried from periods before l^i.

The objective of the problem is to determine a production plan that minimizes the total expected cost, including set-up, production, holding and backlogging costs and satisfying the deterministic and randomly occurring stochastic demands.

3 Computational experiments

To validate and analyze the proposed Mixed Integer Linear Integer Programming model, instances have been defined from the well-known lot sizing instances of Trigeiro et al. [6] by tailoring them according to the settings of our problem. In particular, the probability of occurrence of a stochastic demand in any period $t \subset [l^i, u^i]$ is normally distributed.

The preliminary results show that an increase in the number of stochastic demands lead to an increases of the computational complexity of the model. This finding motivates us to develop heuristic algorithms to solve the problem for large instances.

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