A voter model on adaptive social Networks

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1 Research context and motivation

Nowadays, the study of large-scale network morphogenesis is now well established as a coherent field. On the contrary, temporal network theory is still in its infancy, but gains more and more attention from various research domains, especially in the area of social networks analysis. Indeed, encompassing both the agents *and* the links evolution is a step forward to a better understanding of the subtle network-induced effects arising in large-scale social systems.
In this context, we propose and analyze a voter model (an instance of attractive spin systems, see [1]) with a specific refinement: the interaction graph is *adaptive* in the sense that agents can break links and create new links depending on the spins. Consistent with psycho-social empirical studies and observations, the defined process encapsulates two canonical social behaviours: *selective exposure* and *homophily.* The main original feature of our work lies in the link creation protocol where an agent seeks new friends among its 2-hop neighbours, that is *the friends of his friends.* We completely characterize the absorbing states of such dynamics and give theoretical results on one particular case. Numerical simulations are also provided.
Related literature. In [2], the link creation protocol (LCP) is done by choosing a neighbour uniformly at random (u.r) over the whole graph. In [3], the LCP is different from ours, based on preferential attachment mechanism. In the area of social sciences, homophily and selective exposure are canonical notions that have been well described in the literature: see [4, 5].

2 Model description

Consider a population of K agents. We note $k \in [K] := \{1, ..., K\}$. Each agent k is endowed with a spin $x_k \in \{+1, -1\}$. The spin can represent an orientation, a vote, or a preference. The agents interact throughout a directed unweighted randomly-evolving graph $A(t) \in \{0, 1\}^{K^2}$ with $t \in \mathbb{R}_+$ and $a_{kj}(t) \in \{0, 1\}$. Each agent k is influenced by her out-neighbours $N_k := \{j \in [K] : a_{kj} = 1\}$. We write $k \to j$ if there exists a directed path from k to j, and define a Markov process $(\vec{X}, A)$ whose state-space is $\mathcal{S} := \{+1, -1\}^K \times \{0, 1\}^{K^2}$. Three types of events occur:

- a flip: each agent k picks at rate ϕ uniformly at random (u.r) an agent $j \in N_k$, and if j has a different spin, then k aligns on j. This type of jump is standard in voter models.
- a directed edge gets broken: each agent l picks at rate β u.r an agent $j \in N_l$ and if j has opposite spin, then l breaks the tie. This procedure corresponds to *selective exposure*: the natural trend one has to dismiss dissonant information.
- a directed edge gets created: each agent l picks at rate γ u.r an agent $j \in N_l$, and is looking for new friends. Then, l chooses u.r an agent $m \in N_j$, and if and only if m and l has same spin ($x_l = x_m$), then l gets connected to m provided it was not the case before. The hypothesis « $x_l = x_m$ » can be viewed as a very simple instance of *homophily*: agents create new directed edges much easily toward people who behave alike.

3 Results

Absorbing states. We first characterize the absorbing states. Let

$$\mathcal{A} := \{(\vec{x}, A) \in \mathcal{S} : k \to j \implies \forall l \in \mathcal{C}(k), \forall m \in \mathcal{C}(j), a_{lm} = 1 \text{ and } x_l = x_m\}, \text{ where} \tag{1}$$

$\mathcal{C}(p)$ stands for the strongly connected component of agent $p \in [K]$.

Proposition 1 *The configurations of $\mathcal{A}$ are **the** absorbing states.*

One particular case We study the particular case where a unique agent labeled agent 0 is under social pressure of two stubborn cliques B^+ and B^- of large size: Card(B^+) =Card(B^-) = $K >> 1$. At initial time, $a_{0k}(0) = 1$, for all $k \in B^+ \bigcup B^-$. At all time, $a_{lm} = 1$ for all $(l, m) \in (B^+ \times B^+) \bigcup (B^- \times B^-)$, and $x_j = \sigma 1\ \forall j \in B^\sigma$, $\sigma = \pm$. Furthermore, the two cliques stay totally disconnected: $a_{ml} = a_{lm} = 0\ \forall (l, m) \in B^+ \times B^-$. The last result states that x_0 converges almost surely in finite time toward $\sigma_\infty \in \{+1, -1\}$ and in addition agent 0 gets finally connected with all the agents of the final spin and only with them: $a_{0j} = 1$ for all $j \in B^{\sigma_\infty}$ and $a_{0i} = 0$ for all $i \in B^{-\sigma_\infty}$ in finite time. Nevertheless, depending on the values of the model's parameters (we have computed a bifurcation point at $\frac{\gamma}{\beta} = 3$), the convergence time may be negligible with respect to K or on the contrary agent 0 may stay hesitant during a time of order a power of K (see figure 1).

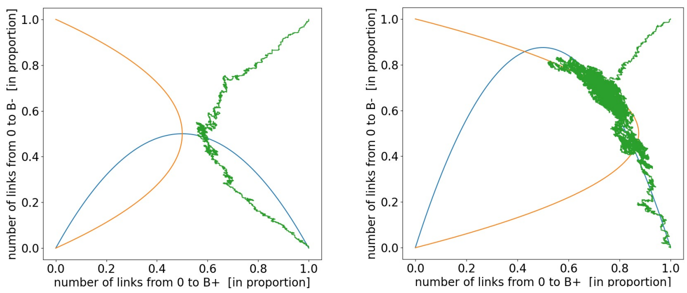

FIG. 1: In green: the trajectory of the normalised neighbourhood N_0. For $\frac{\gamma}{\beta} = 3.5$, persistent hesitation occurs (right). On the contrary, agent 0 is quickly convinced when $\frac{\gamma}{\beta} = 2$ (left). As expected, the trajectory converges towards a full connection of agent 0 to only one clique.

Research perspectives. One may also investigate how a clique of consensual agents $k \in [K]$ reacts facing the arrival of a disagreeing agent 0 getting connected with all of them in the two directions: $a_{lm}(0) = 1\ \forall l, m$, $x_j(0) = +1\ \forall j \neq 0$ and $x_0(0) = -1$.

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