Preprocessing algorithm for the optimization of shortest paths in ecological landscapes

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1 Introduction

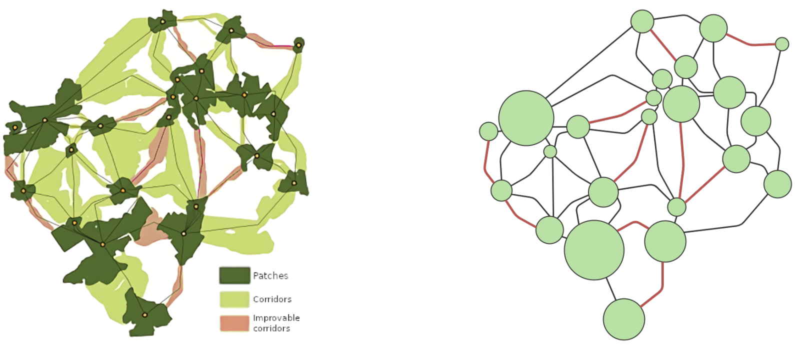
The *landscape connectivity* has been defined by [7] as the degree to which a landscape facilitates the movement of individuals between habitat areas. Beyond the capacity of moving around to access vital resources, landscape connectivity increases gene flow among populations and improves their adaptability to climate change [2, 4].

An ecological landscape can be viewed as a directed graph $G = (V, A, w, l^+)$. Each node $u \in V$ models a portion of the landscape and is associated with a weight w_u that represents its ecological quality, usually the area of potential habitat for a given species. Each arc $(u, v) \in A$ represents a connection that individuals of this species can use to travel from the node u to the node v and the length $l^+(u, v)$ represents the difficulty for an individual to make such travel. With this formalism, ecologists have developed many indicators to quantify the landscape connectivity. We focus on the equivalent connected area indicator – ECA – [6] that can be defined as :

$$ECA(G = (V, A, w, l)) = \sqrt{\Big(\sum_{s,t \in V} w_s \cdot w_t \cdot \exp(-\alpha \cdot d(s, t)) \Big)}$$

where $d(s, t)$ is the length of a shortest path from s to t using $l(u, v)$ as the length of the arc (u, v) and α is a constant specific to the studied species.

The length l^+_a of each arc $a \in A$ can be improved to $l^-_a \leq l^+_a$ by paying a cost c_a. The *Budget Constrained ECA Optimization Problem – BC-ECA-Opt –* consists in finding a subset $S \subseteq A$ of cost at most B that maximizes $ECA(G' = (V, A, w, l'))$ where $l'_a = l^-_a$ if $a \in S$ and $l'_a = l^+_a$ otherwise.



(a) ecological landscape (b) graph representation

2 Contributions

In [3], we have proposed a new generalized flow model that improves existing models for solving BC-ECA-Opt. However, for medium size landscapes with few hundreds of nodes the MIP formulation size exceeds the capacity of the best MIP solvers. This motivates the design of a preprocessing algorithm that reduces the size of the graphs on which generalized flows have to be computed and the study of the following algorithmic problem. Let $\mathcal{L}$ be the set of length functions l such that $l_a \in \{l_a^-, l_a^+\}$ for every arc $a \in A$. Let $d_l(s,t)$ be the length of the shortest st-path with respect to the length function l. An arc (u,v) is said to be *t-strong* (resp. *t-useless*) if $d_l(u,t) = l(u,v) + d_l(v,t)$ (resp. $d_l(u,t) < l(u,v) + d_l(v,t)$) for any of the $2^{|A|}$ length functions $l \in \mathcal{L}$. The problem of identifying strong and useless arcs has been studied as a preprocessing step for other combinatorial optimization problems having as input a graph whose arc lengths may change [1, 5]. In [1] the authors identify a sufficient (but not necessary) condition for a given arc (u,v) to be t-strong (resp. t-useless). A sufficient and necessary condition for an arc (u,v) to be t-useless is given in [5] together with a $O(|V| \cdot (|A| + |V| \log |V|))$ recognition algorithm. Given an oriented graph G, an arc (u,v) and two length functions l^+ and l^-, we provide an $O(|A| + |V| \log |V|)$ algorithm that computes the set of vertices t such that (u,v) is t-strong (resp. t-useless). Our experiments show that preprocessing strong and useless arcs allows to significantly reduce the mixed integer formulation size of BC-ECA-Opt instances. This both speeds up the resolution and allows handling instances with bigger graphs. We hope that this algorithm could be of similar interest for some robust optimization problems as hinted in [1, 5].

Références

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