Linear reformulations for the availability-aware Virtual Network Function placement and routing problem

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1 Introduction

In the context of telecommunication virtualized networks, a Service Function Chain (SFC) is an origin-destination traffic demand having some specific service requirements. These service requirements are expressed as an ordered set of Network Functions (NF) that must be visited – on the given required order – along the SFCs origin-destination route. Firewalls, video optimizers, load balancers, and parental control are standard examples of NFs.

The virtualization of NFs allows the execution on demand of a network function within a virtual server, dissociating it from any dedicated hardware. While the virtualization of network functions allows more flexibility and cost reductions for the service deployment, it also represents a substantial challenge for infrastructure providers since the failure of a single node where a network function is hosted causes the crash of the whole SFC. In order to ensure strict Service Level Agreements (SLAs), a backup plan is required in case some network nodes fail. In other words, the service provider must ensure for each SFC that a set of alternative paths is ready to route it whenever its nominal path is unavailable. Notice that an alternative path can only route a given SFC if the network functions it requires are already placed along the considered the path. Since the placement of network functions incurs extra costs, a relevant combinatorial problem is to choose where to place such functions in order to minimize the costs while ensuring high standards of resilience.

2 Problem definition

We next define the Availability-aware Virtual Network Function Placement and Routing (AVNFPR) problem.

Let $G = (V, A)$ be a directed, loopless, connected graph. Each node $v \in V$ has a capacity $C_v \in \mathbb{R}^+$, and an availability $0 < a_v < 1$, (*i.e.*, a risk of $1 - a_v$ of being down). Moreover, let $\mathcal{F}$ be the set of Virtual Network Function (VNF) types, where each VNF $f \in \mathcal{F}$ has a resource consumption $r^f \in \mathbb{R}^+$, and a placement cost $c_v^f \in \mathbb{R}^+$ for each node $v \in V$. Finally, let K be the set of SFC demands, where each demand $k \in K$ is defined by (i) an origin $o_k \in V$ and a destination $d_k \in V$, (ii) a bandwidth $b_k \in \mathbb{R}^+$, (iii) a required availability $A^k \in [0, 1]$, and (iv) an ordered set of distinct VNFs $F^k \subseteq \mathcal{F}$ that must be visited.

The availability of a path π routing a SFC is defined as the probability that all its VNFs are properly running. If $S \subseteq V$ is the set of nodes hosting a VNF for the given SFC, the path availability is then given by

$$a(\pi) = \prod_{v \in S} a_v.$$

Usually, a single path is not enough for ensuring the SFC's required availability. In this case, a set of paths $\mathcal{P}$ is assigned to the SFC and the availability of such SFC is hence defined as the probability that at least one of its assigned paths is available, that is,

$$a(\mathcal{P}) = 1 - \prod_{\pi \in \mathcal{P}} a(\pi).$$

A set of paths $\mathcal{P}$ is then said to secure SFC $k \in K$ if $a(\mathcal{P}) \geq A^k$.

The AVNFPR problem consists in finding (i) the optimal VNF placement on nodes, and (ii) for each SFC, the associated set of paths passing through the requested VNFs in the right order that secures the SFC. Additionally, node capacities, path latency and arc bandwidth volumes must be verified.

3 Problem formulation and contributions

Notice that for a given SFC $k \in K$, the set of its assigned paths may be exponentially large. This causes the problem of checking the feasibility of a given solution to not belong to NP. Here we study a variant of this general problem that corresponds to a strict SFC security policy. More precisely, we say that two distinct paths assigned to the same SFC cannot use the same node for hosting a VNF. This amounts to say that, for each SFC, the sets of nodes hosting a VNF within a path must be disjoint. The assumption of such policy provides a trivial upper bound of $|V|$ on the number of paths that can be assigned to each SFC.

A natural formulation for the AVNFPR problem can then be obtained using a polynomial number of variables. Due to the page limit, we provide only the availability constraints here below.

$$\prod_{p=1}^{|V|} \left(1 - \prod_{v \in V} \left(\alpha_p^k - (1 - a_v) y_{vp}^k \right) \right) \leq 1 - A^k \qquad \forall k \in K, \tag{1}$$

where y_{vp}^k is a binary variable stating whether or not a VNF is hosted on node $v \in V$ within the p-th path assigned to SFC $k \in K$, and α_p^k is a binary variable stating whether or not the p-th path is used for routing SFC $k \in K$.

Clearly, the presence of the non-linear inequalities (1) makes the formulation difficult to be solved by standard MIP solvers (*e.g.*, CPLEX, Gurobi). In this presentation, we provide some insights on how to linearize such constraints. More precisely, we employ an exponential number of linear constraints to replace the non-linear constraints.

Due to the computational complexity of solving the resulting MILP formulation and inspired by the works from [1, 2, 3], we also explore ways of approximating the problem's feasible region with a polynomial number of constraints. Finally, combinatorial bounds and valid inequalities are also investigated in order to reinforce the proposed formulations.

Références

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