Modeling uncertainty processes in strategic energy planning optimization

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Mots-clés : *Strategic energy planning, Stochastic Dual Dynamic Programming, Auto-regressive process, Markov chain.*

1 Introduction

This paper deals with the modeling of complex stochastic processes in the long-term multistage energy planning problem [1] which is characterized by the presence of many sources of uncertainty, both in the objective function and in the constraints. Starting from basic projections consisting of ranges for the future values of parameters such as demands, energy costs and technology efficiencies, we model the temporal correlation of these uncertain parameters through auto-regressive models. Due to the distinct role played by these parameters in the model, some of them require discretization via Markov chains. The resulting formulation is then solved with an advanced SDDP algorithm available in the literature that handles finite-state Markov chains. Our numerical experiments, performed on the Swiss energy system, show a very desirable adaptation strategy of investment decisions to uncertainty scenarios, a behavior that is not observed when the temporal correlation is ignored. Moreover, the solutions lead to better out-of-sample cost performances than the non-correlated ones which usually yield overcapacities to protect against high, but unlikely, parameter variations over time.

2 Multistage Stochastic Linear Programming

The problems of real world energy systems are usually of a dynamic nature, where uncertain parameters ξ_t are revealed sequentially and decisions must be adjusted to the recent realizations. It can be seen as a two-stage model sequence

$$\min_{(y_1,z_0)\in\mathcal{X}_1(y_0,\xi_1)} f_1(y_1,z_0,\xi_1) + \mathbb{E}_{\xi_2|\xi_1}[Q_2(y_1,\xi_2)] \tag{1}$$

where $\{y_t,z_t\}_{t=1}^T$ describe a policy (a solution to the MSLP) and the recourse term $Q_2(y_1,\xi_2)$ similarly depends on the decisions to be made at later stages with

$$Q_t(y_{t-1},\xi_t) = \min_{(y_t,z_{t-1})\in\mathcal{X}_t(y_{t-1},\xi_t)} f_t(y_t,z_{t-1},\xi_t) + \mathbb{E}_{\xi_{t+1}|\xi_t}[Q_{t+1}(y_t,\xi_{t+1})] \tag{2}$$

for $t=2,...,T$, where the term $Q_{T+1}(y_T,\xi_{T+1})\equiv 0$ at last-stage recourse term $Q_T(y_{T-1},\xi_T)$. To ensure that the model is well defined, we impose relatively complete recourse for any policy y_t,z_t $\forall t=2,...,T$. Functions $Q_t(y_{t-1},\xi_t)$ are referred as *future cost functions* and $\mathcal{Q}_t = \mathbb{E}_{\xi_{t+1}|\xi_t}[Q_{t+1}(y_t,\xi_{t+1})]$ as *expected future cost functions.*

3 SDDP, Autoregressive models and Markov Chains

We solve (1) using the Stochastic Dual Dynamic Programming (SDDP) algorithm in which the stochastic process is stage-wise independent. Despite its advantages, an important limitation of the SDDP is related to the modeling of the dynamics of the state variables. The most common way to preserve dependence within the SDDP is to assume that the stochastic process is modeled as an autoregressive process of order 1 (AR(1)). An AR(1) is incorporated into the SDDP model by defining the stochastic process as a state variable, the noise or innovation ξ_t is considered stage-wise independent and the recursive equation is added to the model. Note that if the new state variable multiplies any other decision variable, bilinear terms originate that destroy the convexity of the future cost function. Therefore, to maintain the mathematical properties that make the method efficient, only the processes (in our case the demands) that are on the right-hand side of the constraints can be expressed as an affine function of the errors.

However, there is another way to preserve the dependence in the SDDP without destroying the convexity of the problem, and that is when the stochastic process is modeled through a finite-state Markov chain. A stochastic process $\{\xi_t\}$ is called Markovian if the conditional distribution of ξ_t given $\xi_{[t-1]}$ is the same as that of ξ_t given ξ_{t-1} for $t = 1, ...T$. When a data process is assumed to be Markovian, a future cost function must be enumerated for each value taken by the state variable, which increases the dimensionality of the problem. It is the main computational disadvantage of this approach. Also, if the process is multi-dimensional, the number of Markov states needed to obtain a good enough solution becomes prohibitive. In this paper, we constructed a Markov chain (homogeneous case) with N-state variables can be derived from an autoregressive process of order 1 to model uncertainty on resource costs. The Markov chain is then introduced in the SDDP formulation.

4 Numerical reuslts

The problem was modeled under three different approaches : the deterministic one, where all input data of the stochastic processes are fixed at nominal value ; the simple SDDP model, where all processes of the random variable are considered stage-wise independent, and finally the MC-SDDP model which combines the SDDP with a Markov chain and introduces additional state variables and constraints into the formulation to model the right side uncertainty by means of a linear time series AR(1).

In our numerical results, the MC-SDDP model proves that considering time dependence can provide benefits in terms of out-of-sample performance. Also the optimal policies of the MC-SDDP protect the system better from extreme events. Therefore the use of better autoregressive models can improve the quality of the results through stochastic optimization. On the other hand, the SDDP model helps to understand the importance of establishing the assumptions and modeling the random variable, since it can leave undesirable solutions. Although the total cost of the two models have a very similar distribution, the non-adaptability of the policy given by the standard SDDP may generate a high risk of overcapacity if future demand is low.

Références

[1] Stefano Moret, Víctor Codina Gironès, Michel Bierlaire and François Maréchal. Characterization of input uncertainties in strategic energy planning models. *Applied Energy*, 202 : 597–617, 2017.