Variantes du polytope min-up/min-down

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Mots-clés : *Polytope, Facettes, Min-up/min-down, Unit Commitment*

1 Introduction

Consider a time horizon $\mathcal{T} = \{1, ..., T\}$ and a set of possible states $\mathcal{S} = \{0, ..., n\}$ for a given system. The system must be in exactly one state at a time. Therefore, using binary variables x_t^i indicating that the system is in state i at time t, the following exclusion constraint holds :

$$\sum_{i \in \mathcal{S}} x_t^i = 1 \qquad \forall t \in \mathcal{T} \tag{1}$$

We introduce generalized min-up/min-down constraints as follows. If the system switches to state $i \in \mathcal{S}$ at time t, then it must remain in state i for at least L^i time periods.
Such constraints can be written :

$$x_{t'}^i \geq x_t^i - x_{t-1}^i \qquad \forall t' \in \{t+1, ..., t + L^i - 1\}, \qquad \forall i \in \mathcal{S} \tag{2}$$

These constraints generalize minimum up and down time constraints from the literature [1, 2] in the sense that the system has an arbitrary number n of possible states, instead of only two states (up and down).
Generalized min-up/min-down constraints appear in practical Unit Commitment Problems, where nuclear and hydro production units have discrete production levels.
For an n-state system, we define the generalized min-up/min-down polytope as follows :

$$P_n(x) = conv\Big\{x \in \{0,1\}^{(n \times T)} \mid (1),\ (2)\Big\}$$

In this paper, we study polytope $P_n(x)$ in an extended variable space. We also study a variant of $P_n(x)$ featuring precedence constraints instead of exclusion constraints.

2 Related work

For a system with only 2 possibles states (up and down, or equivalently 0 and 1), a complete description of $P_2(x)$ is given in [1], using an exponential number of inequalities.
The authors of [2] introduce binary variables u_t, indicating a switch from state 0 to state 1. They prove that along with trivial inequalities, the following inequalities give a complete description of the min-up/min-down polytope in the variable space (x, u) :

$$\left\{\begin{array}{lll} \displaystyle\sum_{t'=t-L^1+1}^{t} u_{t'} \leq x_t^1 & \forall t \in \{L^1+1, ..., T\} & \text{(3a)} \\ \displaystyle\sum_{t'=t-L^0+1}^{t} u_{t'} \leq 1 - x_{t-L^0}^1 & \forall t \in \{L^0+1, ..., T\} & \text{(3b)} \\ u_t \geq x_t^1 - x_{t-1}^1 & \forall t \in \{2, ..., T\} & \text{(3c)} \end{array}\right.$$

3 Polyhedral study

Generalized min-up/min-down polytope Extending the definition of variables u to an n-state system, we define polytope

$$P_n(x,u) = conv\left\{(x,u) \in \{0,1\}^{(n\times T)\times(n\times T-1)} \mid (1),\ (2),\ u_t^i \geq x_t^i - x_{t-1}^i\right\}$$

where variable u_t^i indicates that state i is switched on at time t.

We show that generalizations of inequalities (3) are facets of $P_n(x,u)$. Moreover, we introduce a family of facets arising from the coupling between (1) and (2). We show that this set of inequalities give a complete description of $P_n(x,u)$.

Generalized min-up/min-down polytope with precedence constraints We consider a variant where exclusion constraints (1) are replaced with the following precedence constraints :

$$x_t^i \geq x_t^{i+1} \qquad \forall t \in \mathcal{T}, \qquad \forall i \in \{0,...,n-1\} \tag{4}$$

In other words, the states of the system are incremental, in the sense that if state i is on at time t, then all states $j < i$ must also be on at time t.

Note that in this case, the min-up/min-down constraints (2) have another meaning. Namely they enforce that when state i is switched on, the system must remain on states $j \geq i$ during L^i time steps.

We define the precedence constrained generalized min-up/min-down polytope :

$$P_n^{prec}(x,u) = conv\left\{(x,u) \in \{0,1\}^{(n\times T)\times(n\times T-1)} \mid (4),\ (2),\ u_t^i \geq x_t^i - x_{t-1}^i\right\}$$

We introduce two polynomial families of facets for $P_n^{prec}(x,u)$, one that generalizes (4) using u variables, and another that captures the coupling between (3) and (4). We also discuss polyhedral characterization.

Références

[1] J. Lee, J. Leung, and F. Margot. Min-up/min-down polytopes. *Discrete Optimization*, 1 :77–85, 2004.

[2] D. Rajan and S. Takriti. Minimum up/down polytopes of the unit commitment problem with start-up costs. *IBM Research Report*, 2005.