The roll-out of new mobile technologies as a timing game

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1 Introduction

In the telecommunication industry, the roll-out of a new mobile communication technology is a challenge faced periodically by mobile operators which are required to upgrade their infrastructure. In this paper, we consider the problem of determining the best strategy for the introduction of 5G on the market operated by telco operators. We focus especially on ensuring the most efficient timing to perform such upgrades on the network. Operators seek to delay the deployment costs, taking into account that infrastructure upgrades by other operators might let customers switch to such competitors.

We formulate the problem in the form of a discrete-time timing game and we introduce game-theoretical solution concepts to determine the optimal choice in a competitive setting. We take into account logistical and normative constraints, set by the regulator. Both operators can track each other's 5G deployment, by observing the evolution of the quality of service, which is a piece of public information.

2 Model

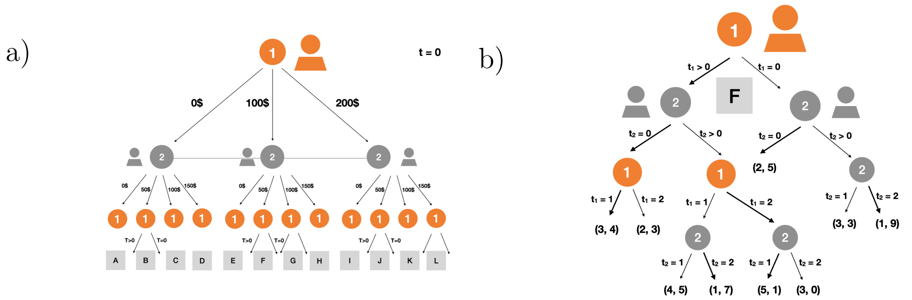
In our model we consider two telecommunication operators, whose objective is to optimize their strategy. Each operator chooses the subsidy she offers to the customers from a discrete set S_i. Afterwards, they schedule when and where to deploy investments on their own *sites*. Operators act on a discrete time horizon. We thus introduce the following parameters : $N = \{1, 2\}$, set of players ; S_i, set of possible subsidies for player $i \in N$ chosen at time $t = 0$; $T = \{1, ..., |T|\}$, set of time-intervals over which operators act to install the new technology ; $\mathcal{A}$, set of sites.

We introduce $z_{i,a,t} \in \{0, 1\}$, a binary variable indicating if the new technology is installed on site a by operator i at time $t \geq 1$. We call $t_{i,a}$ the time at which operator i installs it on site a. The operators' schedule is bounded by some constraints :

- *Logistic constraints :* the operator i can invest on a limited number Z_i of sites at each time $t \geq 1$. Thus for every player it holds $\sum_{a\in\mathcal{A}}(z_{i,a,t+1} - z_{i,a,t}) \leq Z_i$;
- *Regulator constraints :* before every time t at least $R(t)$ sites have to support the new technology. Thus for all players and for all $t \geq 1$ it holds $\sum_{a\in\mathcal{A}} z_{i,a,t} \geq R(t)$.

In order to identify a solution we add a further assumption, i.e. that players can observe, at time t, the history of actions taken by the other player for $t' < t$, and react accordingly ; the choice of the opponent's subsidy is observable by both players at time $t = 0$.

A convenient model that complies to such assumption is that of a *game in extensive form* [1], whose mathematical model is based on a *game tree*. Every combination of chosen strategies $\{(\mathbf{s}_i, \mathbf{z}_i), i = 1, 2\}$ leads to an *outcome*. The *utility function* evaluates such outcomes for every



c)

		Operator 2			
		0\$	50\$	100\$	150\$
Operator 1	0\$	A	B	C	D
	100\$	E	F	G	H
	200\$	I	J	K	L

FIG. 1 – a) Subsidies are picked first. b) After the subsidies are decided, players choose sequentially to add or not the new technology on the site at each round $t \in T = \{1, 2, \ldots, |T|\}$. c) Matrix representation of the first part of the game.

operator ; it increases with their market share and it decreases with the costs incurred for the technology upgrades and their promotion. The higher the value of u_i, the higher the value a player assigns to an outcome. For our system the *game tree* is represented in Figure 1a) and b) and defined below.

Définition 1 (Service providers (SP) game) *The service providers game* $\langle N, S_1, S_2, \mathcal{A}, T, u\rangle$ *is an extensive form game with* $N = 2$ *players* $\{1, 2\}$ *competing over set of sites* $\mathcal{A}$ *in which :*
- *at the root vertex both players choose at the same time* $t = 0$ *and independently the subsidies* $s_1 \in S_1$ *and* $s_2 \in S_2$ *;*
- *the players act in sequence at every round* $t \geqslant 1$*, starting from player* 1*. At every step they can decide on which sites* $A_{1t} \subseteq \mathcal{A}$ *and* $A_{2t} \subseteq \mathcal{A}$ *install the new technology, given the constraints ;*
- *after* $|T|$ *rounds the game ends and the actions chosen at each round are evaluated for every player* i *by the utility functions* $u_i : S_1 \times S_2 \times (\mathcal{A} \times T)^2 \to \mathbb{R}$.

The solution of the game can be determined in two steps. First, at $t = 0$ the operators pick a subsidy at the same time. The second operator does not know what the first operator has played, and vice versa. Then, in the second part (all subtrees rooted at $t = 1$), they get to know what the other operator has chosen and decide one after another if installing on a site or not at each time step, starting by player 1. Let us denote with $\Gamma(s_1, s_2)$ the part of the game which starts from $t = 1$, given that the first operator has chosen $s_1 \in S_1$ and the second operator $s_2 \in S_2$. If no player has the same payoffs at two different outcomes, such game has a unique *subgame perfect equilibrium* [1], i.e. a Nash equilibrium [2] for every subgame (which is represented by a subtree). Different choices of subsidies can lead to different timings of the investment and thus different outcomes. We define the matrix M which maps a couple of choices for the subsidies to the utility of such outcome $(s_1, s_2) \to \sigma(s_1, s_2) \to M(s_1, s_2) = u(s_1, s_2, \sigma(s_1, s_2))$. Matrix M defines a game, which has at least one Nash equilibrium [2]. Operators pick the optimal combination of strategies within the set of such Nash equilibria.

Définition 2 *Given a SP game* $\langle N, S_i, \mathcal{A}, T, u\rangle$ *and its correspondent matrix* $M : (s_1, s_2) \mapsto u(s_1, s_2, \sigma(s_1, s_2))$*, with* (s_1, s_2) *chosen at time* $t = 0$ *and* $\sigma(s_1, s_2) \in (\mathcal{A} \times T)^2$ *the optimal installation times chosen at times* $t \geq 1$*, we say* $(\bar{s}_1, \bar{s}_2) \in S_1 \times S_2$ *is an equilibrium if for all* $s_1 \in S_1$ *and* $s_2 \in S_2$ *we have :* $M_1(\bar{s}_1, \bar{s}_2) \geq M_1(s_1, \bar{s}_2)$, $M_2(\bar{s}_1, \bar{s}_2) \geq M_2(\bar{s}_1, s_2)$.

Références

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