

# Optimizing the makespan service level for the stochastic flexible job shop scheduling problem

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## 1 Introduction

The Stochastic Flexible Job-shop Scheduling Problem (SFJSP) is an extension of the FJSP, in which a set of jobs have to be processed on a collection of machines, and each job requires a number of consecutive operations (routing) before being completed. Finding a solution to this problem is to determine both an assignment of operations to machines, and the sequence the operations must follow, while respecting the routing of every job.

The SFJSP has been considerably less studied than its deterministic version. In the FJSP, all problem parameters are supposed to be known, and set-up times between operations to be negligible or included in the processing times. In the studied SFJSP, processing times are considered unknown and characterized by independent random variables following known finite probability distributions. Under random processing times and for a given makespan, the probability that the makespan is lower than or equal to a given threshold is maximized. To solve this problem, a solution approach, based on tabu search embedding a Monte Carlo simulation to deal with uncertainties, is introduced in our previous work [2]. In this abstract, we present complementary numerical results comparing the sequences obtained by the proposed method in [2] with respect to worst-case and best-case scenarios.

## 2 Problem Statement and Solution Approach

In the studied SFJSP, the processing times are random variables. As a consequence, the makespan of a sequence is also a random variable. To deal with a random makespan, we are interested in maximizing the probability that makespan  $C_{max}$  is lower than or equal to a given threshold, as proposed in [2]. This probability, denoted by  $\alpha(S, T)$ , is known as the *makespan service level* associated to sequence  $S$  and threshold  $T$  :

$$\alpha(S, T) = \mathbb{P}(C_{max}(S) \leq T).$$

The proposed solution method is based on a competitive tabu search approach [1], including a Monte Carlo sampling procedure to represent and deal with uncertainties. To determine the service level of a sequence, a set of scenarios  $\Omega$  is generated and an algorithm based on Monte Carlo simulation is implemented, as described in [2].

In this abstract, we compare the results of the deterministic FJSP corresponding to the best-case and worst-case scenarios with the results obtained in [2] to show the relevance of the makespan service level.

### 3 Computational Experiments

The theoretical and empirical worst-case (resp. best-case) scenarios are considered. The theoretical worst-case scenario  $\omega_{worst}^{th}$  (resp.  $\omega_{best}^{th}$ ) corresponds to the maximal (resp. minimal) value in the definition domain of random variables. In the empirical worst-case scenario  $\omega_{worst}^{emp}$  (resp.  $\omega_{best}^{emp}$ ), the value for every random processing time is the largest (resp. smallest) among  $\omega \in \Omega$ . For each scenario  $\omega \in \Omega$  and for every operation  $i$  and every machine  $j$  ( $\omega_{ij}$ ), we have :

$$\omega_{ij} \leq \omega_{worst,i,j}^{emp} \leq \omega_{worst,i,j}^{th}$$

and

$$\omega_{best,i,j}^{th} \leq \omega_{best,i,j}^{emp} \leq \omega_{ij}$$

TAB. 1 – Makespan and *makespan service level* for every job from instance mt10-vdata of [3].  $T = 655$ .  $\alpha = \alpha(S, T, \Omega)$

Job	$S_\mu, \omega_{worst}^{emp}$		$S_{best}^{emp}, \omega_{best}^{emp} - S_{worst}^{emp}, \omega_{best}^{emp}$		$S^*, \bar{p}$	
	$C_{max}(S, \omega)$	$\alpha$	$C_{max}(S, \omega)$	$\alpha$	$C_{max}(S, \omega)$	$\alpha$
1	744.20	0.932	655.00 - 655.93	0.050 - 1	655	1
2	743.72	0.935	655.00 - 655.20	0.001 - 1	655	1
3	768.69	0.979	655.00 - 723.74	0.805 - 0	655	1
4	880.86	0.905	597.00 - 834.93	0.958 - 0	655	0.978
5	702.62	0.856	655.00 - 655.00	0.473 - 1	655	1
6	724.30	0.985	655.00 - 656.00	0.002 - 1	655	0.998
7	690.87	0.953	655.00 - 655.56	0 - 1	655	1
8	778.63	0.864	655.00 - 685.93	0 - 0	655	0.898
9	841.38	0.863	655.00 - 748.84	0 - 0	655	0.956
10	751.98	0.919	655.00 - 691.99	0.761 - 0	655	0.989

Solving the FJSP for the best-case and worst-case scenarios with a tabu search approach does not bound the problem at hand. However, it provides interesting information on the possible outcome of both the makespan and the makespan service level.

### 4 Conclusions

This paper focuses on the stochastic flexible job shop scheduling problem under random processing times. Based on the extended benchmark instances of the FJSP, extensive computational experiments were conducted to evaluate the performance of the proposed approach and to highlight the relevance of the concept of makespan service level.

### Références

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