Polyhedral Investigation and Branch-and-Cut Algorithm for the Spectrum Assignment Problem *

Ibrahima Diarrassouba[1], Youssouf Hadhbi[2] , Ali Ridha Mahjoub[3]

[1] Le Havre Normandie University, LMAH, FR CNRS−3335, 76600 Le Havre, France
diarrasi@univ-lehavre.fr

[2] Clermont Auvergne University, LIMOS, UMR CNRS−6158, 63178 Clermont Ferrand, France
youssouf.hadhbi@uca.fr

[3] Paris Dauphine-PSL University, LAMSADE, UMR CNRS−7243, 75775 Paris CEDEX 16, France
ridha.mahjoub@lamsade.dauphine.fr

Keywords : *Network design, routing, spectrum assignment, integer programming, polyhedron, facet, valid inequalities, symmetry-breaking, separation, branch-and-cut, primal heuristic.*

1 Introduction

In this work we focus on the Spectrum Assignment (SA) problem related to the dimensioning and designing of Spectrally Flexible Optical Networks (SFONs). It can be stated as follows. Consider a SFON as an undirected, loopless, and connected graph $G = (V, E)$, and an optical spectrum $\mathbb{S} = \{1, \dots, \bar{s}\}$ of available frequency slots. Let K be a multiset of demands such that each demand k is specified by an origin node $o_k \in V$, a destination node $d_k \in V \setminus \{o_k\}$, a slot-width $w_k \in \mathbb{Z}_+$, and a routing path p_k from its source o_k to its destination d_k through G. The SA consists of determining for each demand $k \in K$ an interval of contiguous frequency slots $S_k \subset \mathbb{S}$ of width equal to w_k (continuity and contiguity constraints) such that $S_k \cap S_{k'} = \emptyset$ for each pair of demands $k, k' \in K$ ($k \neq k'$) with paths sharing an edge , i.e., $E(p_k) \cap E(p_{k'}) \neq \emptyset$ (non-overlapping constraint), while optimizing the number of slots allocated in $\mathbb{S}$.
The SA is well known to be NP-hard problem [1]. It is equivalent to the problems of wavelength assignment, interval coloring, and dynamic storage allocation [1] that are well known to be NP-hard. To the best of our knowledge, a polyhedral approach to the SA problem has not been considered before, even to its equivalent problems. The main aim of our work is to provide a deep polyhedral investigation and design a cutting plane method for the problem. First, we propose an integer linear programming compact formulation and investigate the facial structure of the associated polytope. Moreover, we identify several classes of valid inequalities for the polytope and prove that these inequalities are facet-defining. We further discuss their separation problems. Based on these results, we devise a Branch-and-Cut (B&C) algorithm for the problem.

2 ILP Formulation

We first introduce an ILP compact formulation. For this, we consider for each $s \in \mathbb{S}$, a binary variable u_s which takes 1 if the slot s is used and 0 if not, and for $k \in K$ and $s \in \mathbb{S}$, let z_s^k be a variable which takes 1 if slot s is the last slot allocated for the routing of demand k and 0 if not. The contiguous slots $s' \in \{s - w_k + 1, ..., s\}$ should be assigned to demand k whenever $z_s^k = 1$. The SA is equivalent to the following integer linear programming problem

$$\min \sum_{s \in \mathbb{S}} u_s, \tag{1}$$

*This work was supported by the French National Research Agency grant ANR-17-CE25-0006.

subject to

$$z_s^k = 0, \quad \text{for all } k \in K \text{ and } s \in \{1, ..., w_k - 1\}, \tag{2}$$

$$\sum_{s=w_k}^{\bar{s}} z_s^k = 1, \quad \text{for all } k \in K, \tag{3}$$

$$\sum_{k \in K_e} \sum_{s'=s}^{\min(\bar{s}, s+w_k-1)} z_s^k - u_s \leq 0, \quad \text{for all } e \in E \text{ and } s \in \mathbb{S}, \tag{4}$$

$$u_s - \sum_{k \in K} \sum_{s'=s}^{\min(s+w_k-1, \bar{s})} z_{s'}^k \leq 0, \quad \text{for all } s \in \mathbb{S}, \tag{5}$$

$$z_s^k \geq 0, \quad \text{for all } k \in K \text{ and } s \in \mathbb{S}, \tag{6}$$

$$0 \leq u_s \leq 1, \quad \text{for all } s \in \mathbb{S}, \tag{7}$$

$$z_s^k \in \{0, 1\}, \quad \text{for all } k \in K \text{ and } s \in \mathbb{S}, \tag{8}$$

$$u_s \in \{0, 1\}, \quad \text{for all } s \in \mathbb{S}. \tag{9}$$

where $K_e = \{k \in K, e \in E(p_k)\}$ denotes the set of demands in K with paths passing through the edge e. Based on this formulation, we conduct an investigation of the polytope defined by the convex hull of all its solutions.

3 Valid Inequalities and Facets

We identify several classes of valid inequalities to obtain tighter LP bounds. Some of these inequalities are obtained by using conflict graphs related to the problem : clique inequalities, odd-hole and lifted odd-hole inequalities, and also cover-based inequalities related to some capacity constraints. We also give necessary and sufficient conditions under which these inequalities are facet defining. We further study the related separation problems and devise separation algorithms for these inequalities.

4 Branch-and-Cut Algorithm

Using the polyhedral results and the separation algorithms, we devise a B&C algorithm to solve the problem. To boost the effectiveness of the B&C algorithm, we propose some enhancements based on a warm-start algorithm, and primal-heuristic to obtain tighter primal bounds. On the other hand, we have noticed also that several symmetrical solutions may appear given that there exist several feasible equivalent solutions that have the same value, and they can be found by doing some permutations between the slots assigned to some demands while satisfying the SA constraints. For that, we derive some symmetry-breaking inequalities for the SA in order to well manage the equivalent sub-problems in the B&C tree. Moreover, we provide some lower bounds obtained by using some properties of the conflict graph. Based on all this, we present an extensive experimental study while showing the impact of the valid inequalities and symmetry-breaking inequalities on the effectiveness of the B&C algorithm using two types of instances : random and realistic ones with $|K|$ up to 300 and $\bar{s}$ up to 320. They are composed of two types of graphs (topologies) : real graphs and realistic ones from SND-LIB with $|V|$ up to 161 and $|E|$ up to $|166|$.

Références

[1] Bermond, J.C., Moataz, F.Z. : On spectrum assignment in elastic optical tree-networks. In : Discrete Applied Mathematics Journal 2019, pp. 40-52.