A branch-price-and-cut approach for the Multi-Commodity two-echelon Distribution Problem

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1 Problem definition

The *Multi-Commodity two-echelon Distribution Problem* (MC2DP) [3] considers a two-echelon distribution system composed of a set of suppliers $\mathcal{S}$, a set of distribution centres $\mathcal{D}$ and a set of customers $\mathcal{C}$, where collection and delivery operations are performed. Specifically, $|\mathcal{K}|$ commodities are collected from the suppliers, sent to the distribution centres for consolidation purposes and delivered to the customers to fulfill their requests. For each commodity $k \in \mathcal{K}$, each supplier $i \in \mathcal{S}$ provides an amount $P_{ik} \geq 0$ of k, and each customer $j \in \mathcal{C}$ has a request $R_{jk} \geq 0$ for k. The collection operations are performed by an unlimited fleet of homogeneous vehicles of capacity Q^1 with direct trips from the suppliers to the distribution centres. Conversely, each distribution centre owns an unlimited fleet of homogeneous vehicles of capacity Q^2 performing routes to deliver the commodities to the customers. All vehicles can transport any set of commodities as long as their capacity is not exceeded. In addition, as in the *Commodity constrained Split Delivery Vehicle Routing Problem* (C-SDVRP) (see [1]), customers can be visited multiple times. However, the request for a given commodity has to be delivered in a single visit. The aim of the MC2DP is to fulfill the customer requests not exceeding the vehicle capacities, and the available commodity amounts at the suppliers and such that the overall transportation cost is minimized.

2 Problem formulation and solution method

We define the MC2DP on a directed weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V} = \mathcal{S} \cup \mathcal{D} \cup \mathcal{C}$ and $\mathcal{A} = (\mathcal{S} \times \mathcal{D}) \cup (\mathcal{D} \times \mathcal{S}) \cup (\mathcal{D} \times \mathcal{C}) \cup (\mathcal{C} \times \mathcal{D})$. Each arc $(i, j) \in \mathcal{A}$ is associated with a non-negative cost C_{ij}. For each distribution centre $o \in \mathcal{D}$, set $\mathcal{R}_o$ contains the feasible routes that a vehicle owned by o can perform, i.e., the non-empty cycles in the delivery echelon starting and ending at o such that the total amount of delivered commodities does not exceed vehicle capacity Q^2. We denote by C_r the cost associated with route r and by a^r_{jk} a binary parameter taking value 1 if route r delivers commodity k to customer j and 0 otherwise We introduce the following variables. For all $o \in \mathcal{D}$ and $i \in \mathcal{S}$, integer variable x_{oi} represents the number of vehicles traversing arc (o, i). For all $o \in \mathcal{D}$, $i \in \mathcal{S}$ and $k \in \mathcal{K}$, non-negative continuous variable q^k_{oi} stores the amount of commodity k loaded at supplier i and sent to distribution centre o. Finally, for all $o \in \mathcal{D}$ and $r \in \mathcal{R}_o$, binary variable λ_r takes value 1 if route r is selected in the solution and 0 otherwise. The Master Problem (MP) reads

$$\min \sum_{(o,i)\in\mathcal{A}^1} (C_{oi} + C_{io})x_{oi} + \sum_{o\in\mathcal{D}} \sum_{r\in\mathcal{R}_o} C_r \lambda_r \quad (1)$$

$$\sum_{o\in\mathcal{D}} q^k_{oi} \leq P_{ik} \qquad \forall i \in \mathcal{S}, \forall k \in \mathcal{K} \quad (2)$$

$$\sum_{k\in\mathcal{K}} q^k_{oi} \leq Q^1 x_{oi} \qquad \forall o \in \mathcal{D}, \forall i \in \mathcal{S} \quad (3)$$

$$\sum_{o\in\mathcal{D}} \sum_{r\in\mathcal{R}_o} a^r_{jk} \lambda_r \geq 1 \qquad \forall j \in \mathcal{C}, \forall k \in \mathcal{K} \text{ s.t. } R_{jk} > 0 \quad (4)$$

$$\sum_{i\in\mathcal{S}} q^k_{io} \geq \sum_{r\in\mathcal{R}_o} \sum_{j\in\mathcal{C}} R_{jk} a^r_{jk} \lambda_r \qquad \forall o \in \mathcal{D}, \forall k \in \mathcal{K} \quad (5)$$

$$x_{oi} \in \mathbb{Z}_{\geq 0}, \forall o \in \mathcal{D}, \forall i \in \mathcal{S} \quad q^k_{oi} \in \mathbb{R}_{\geq 0}, \forall o \in \mathcal{D}, \forall i \in \mathcal{S}, \forall k \in \mathcal{K} \quad \lambda_r \in \{0,1\}, \forall r \in \mathcal{R}_o, \forall o \in \mathcal{D}. \quad (6)$$

Objective function (1) minimizes the overall transportation cost. Constraints (2) guarantee that the amount of each commodity available at each supplier is respected. Constraints (3) ensure that a sufficient number of vehicles performs the collection operations. Constraints (4) are the *covering constraints* ensuring that customer requests are fulfilled. Constraints (5) impose the load synchronization strategy specified by commodity to link the collection and delivery echelons at the distribution centres. Constraints (6) define the variables.

To solve the MP, we design a branch-price-and-cut (BPC) algorithm, where, at each column generation iteration, we price λ_r variables. Specifically, we solve a pricing problem $\min\{\bar{C}_r \mid r \in \mathcal{R}_o, o \in \mathcal{D}\}$, where $\bar{C}_r$ is the reduced cost of λ_r, by decomposing it per distribution centre. Each problem $\mathcal{P}_o \coloneqq \min\{\bar{C}_r \mid r \in \mathcal{R}_o\}$ is an *Elementary Shorthest Path Problems with Resource Constraints* (ESPPRC) (see [2]). Our BPC incorporates several classical and advanced speed-up techniques ([4]): the *ng-path* relaxation, the bidirectional labeling search, the automatic dual pricing smoothing stabilization, a multi-phase strong branching procedure and three heuristic approaches to solve the ESPPRC. Two of these are similar to the ones proposed in [2]. The third one is a novel two-phase heuristic. The first phase computes a lower bound on the value of $\mathcal{P}_o$ and a set of promising customer sequences by solving the ESPPRC on a modified graph, where customers are delivered with their least requested commodity. The second phase retrieves the routes to be inserted in the MP by solving the ESPPRC again on acyclic graphs, one for each customer sequence computed in the first phase. Finally, before starting the branching phase, we look for violated valid inequalities. We consider the capacity constraints and two new families of valid inequalities based respectively on the set covering polytope and on the number partitioning problem polytope.

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