Reformulation for a two-stage robust facility location problem

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1 Introduction

1.1 Problem statement

Facility Location Problems (FLPs) are among the most prominent applications of operations research. This type of problem consists, for a company, in deciding the locations for opening facilities so as to serve a given set of clients with maximum efficiency. The vast literature dedicated to FLPs attest that a lot of variants can be considered given this definition.

In this paper, we study FLPs in which the demand of each client is not completely known at decision time, as it typically happens in many practical applications. Moreover, the timing of the taken decisions (i.e., opening facilities and serving clients) suggests a two-stage nature of the decision flow. As a consequence, we consider the case where only the opening of facilities shall be decided here and now, the actual assignment of clients to opened facilities being postponed at a later instant.

Formally, let V_1 be a set of candidate locations for opening facilities and let V_2 be a set of clients. Every location $i \in V_1$ is associated to a setup cost f_i which must be paid for opening a facility in site i as well as a maximum capacity q_i restricting the amount of goods leaving the site. To every client $j \in V_2$, we associate a random variable d_j modeling the demand of j. Additionally, we let p_j denote the unitary profit earned by the company for delivering one unit of product to a client. As it often happens in practical applications, clients require to be entirely served by the same company, and thus, partially serving a client is not rewarded. For every connection $(i, j) \in V_1 \times V_2$, we note t_{ij} the unitary transportation cost from i to j.

1.2 Two-stage robust modeling

As anticipated, the considered problem embeds a two-stage decision flow with uncertain input parameters (i.e., the demands). We therefore propose a two-stage robust formulation of this problem. Following the guidelines of [1], we assume that each client's demand follows a symmetric distribution with mean equal to the nominal value $\bar{d}_j$ in the interval $[\bar{d}_j - \tilde{d}_j, \bar{d}_j + \tilde{d}_j]$. Since it is unlikely that all clients change their demands, we assume that only up to Γ do so where $\Gamma \in \mathbb{N}$ is an input parameter used to control the robustness of the solution. As such, we assume that the worst-case scenario is characterized by two subsets $L \subseteq V_2$ and $H \subseteq V_2$ with $|L| + |H| \le \Gamma$ and the demand of each client $j \in L$ (resp. $j \in H$) is $\bar{d}_j - \tilde{d}_j$ (resp. $\bar{d}_j + \tilde{d}_j$), all remaining clients (i.e., in $V_2 \backslash (L \cup H)$) having a demand equal to $\bar{d}_j$. We introduce set U, which we refer to as the *uncertainty set*, defined as follows.

$$U = \{(L, H) : L \subseteq V_2, H \subseteq V_2, L \cap H = \emptyset, |L| + |H| \le \Gamma\} \tag{1}$$

We model the here-and-now decisions by introducing $|V_1|$ binary decision variables such that, for $i \in V_1$, $x_i = 1$ iff a facility is opened in site i. We note $X = \{0, 1\}^{|V_1|}$ and refer to it as the *here-and-now feasible space*.

The wait-and-see (assignment) problem is modeled as follows: for every client $j \in V_2$, we introduce a binary variable y_j which is set to 1 iff client j is entirely served and, for every connection (i, j) with $i \in V_1$, a non-negative continuous variable s_{ij} representing the amount of goods transported from i to j. Each client j is entirely served if the amount of goods arriving to j is equal to its demand $\hat{d}_j$, as described by constraints (2). Moreover, constraints (3) impose that the total amount of goods leaving a site, say $i \in V_1$, must not exceed its total capacity, q_i. Therefore, for a fixed here-and-now decision $\boldsymbol{x} \in X$ and for given L and H (as described above), we introduce set $Y(\boldsymbol{x}, L, H)$, denoted as the *wait-and-see feasible space*, that includes all vectors $(\boldsymbol{y}, \boldsymbol{S}) \in \{0, 1\}^{|V_2|} \times \mathbb{R}_+^{|V_1|\times|V_2|}$ fulfilling the following constraints:

$$\sum_{i\in V_1} s_{ij} \geq \bar{d}_j y_j + \begin{cases} -\tilde{d}_j y_j & \text{if } j \in L \\ +\tilde{d}_j y_j & \text{if } j \in H \end{cases} \qquad \forall j \in V_2 \tag{2}$$

$$\sum_{j\in V_2} s_{ij} \leq q_i x_i \qquad \forall i \in V_1 \tag{3}$$

Finally, the two-stage robust facility location problem which we consider is the following, with F defined in (5).

$$\min_{\boldsymbol{x}\in X} \left\{ \sum_{i\in V_1} f_i x_i + \max_{(L,H)\in U} \min_{(\boldsymbol{y},\boldsymbol{S})\in Y(\boldsymbol{x},L,H)} F(\boldsymbol{y}, \boldsymbol{S}, L, H) \right\} \tag{4}$$

$$F(\boldsymbol{y}, \boldsymbol{S}, L, H) = \sum_{(i,j)\in V_1\times V_2} t_{ij} s_{ij} - \sum_{j\in L} p_j(\bar{d}_j - \tilde{d}_j) y_j - \sum_{j\in V_2\setminus(S_1\cup S_2)} p_j \bar{d}_j y_j - \sum_{j\in H} p_j(\bar{d}_j + \tilde{d}_j) y_j \tag{5}$$

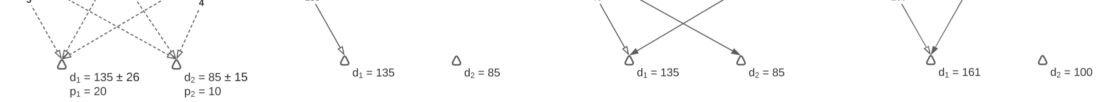
Thus, the problem which we are dealing with is a two-stage robust problem with mixed-integer wait-and-see decision space, constraint uncertainty and discrete uncertainty set. This class of problems is typically hard to solve and no satisfying exact approach has emerged in the literature, see [4] for a recent survey on two-stage robust optimization. Our main contribution is to develop a non-trivial reformulation for this problem which can exactly be solved by exploiting recent advances on cost-uncertain two-stage robust problems.

In the following sub-section, we discuss the practical relevance of our problem by analysing a simple numerical example. In Section 2, we theoretically derive a valid reformulation of our problem where the uncertainty no longer interferes within the constraints. We report early computational results in Section 3.

Remark 1. *In* (2)*, a greater-or-equal sign is used instead of an equality sign. This can be done since profits are not related to the s_{ij}-variables but directly depend on the clients demand.*

1.3 A small example

Le us consider a simple, yet enlightning, example, so as to show the practical interests of our work. We consider the network presented in Figure (1a) where the arcs weight denote the unitary transportation costs between sites (circles) and clients (triangles).



(a) FLP instance (b) $\Gamma = 0$ (c) $\Gamma = 1$ (d) $\Gamma = 2$

FIG. 1: Robust solutions for FLP with different uncertainty budgets Γ

Assuming that no client changes his demand (i.e., $\Gamma = 0$), the optimal solution is to open only one facility in site 1 in order to serve client 1. The associated profit is 695, as it can be seen in Figure (1b) where figures on the arcs are the amount of goods being transported. Yet, it is easily seen that, if client 1 increases his demand to 161 units (i.e., $135 + 26$), the company will not be able to serve this client anymore. Moreover, the company would not be able to switch to client 2 as a reaction, since serving 2 from site 1 would not be profitable.

What can be done, however, is taking into account the demands uncertainty when designing the company's network. For instance, assuming that up to one client may change his demand (i.e., $\Gamma = 1$). Under this assumption, the optimal here-and-now decision is to open two facilities (1 and 3) so as to prevent the previously considered scenario (see Figure (1c)). In such design, the worst scenario which could occur is that no client changes his demand, and the optimal reaction is also depicted in Figure (1c). Note that, with this new design, an increased demand for client 1 can easily be dealt with. The worst-case profit it 190.

In Figure (1d), we depicted the optimal here-and-now decision as well as the optimal wait-and-see decision in the worst case assuming that up to two clients change their demand. The worst-case is one where both two clients increase their demand and the associated profit is 55.

This small example shows the importance and practical relevance of considering robust approaches as sensitivity analysis for network designs in FLP contexts.

2 Theoretical results

In this section, we reformulate problem (4) so as to exploit recent advances on cost-uncertain two-stage robust problems to derive the first exact method for our two-stage robust FLP.

To ease our exposure, let us first introduce a binary set $\Xi \times \{0,1\}^{|V_2|} \times \{0,1\}^{|V_2|}$ comprising all couples of vectors $\boldsymbol{l}$ and $\boldsymbol{h}$ such that the following constraints are satisfied.

$$\sum_{j\in V_2}(l_j + h_j) \le \Gamma \text{ and } l_j + h_j \le 1 \quad \forall j \in V_2 \tag{6}$$

Informally, $(\boldsymbol{l}, \boldsymbol{h})$ now encodes sets L and H as binary decision variables, and one can consider $Y(\boldsymbol{x}, \boldsymbol{l}, \boldsymbol{h})$ equivalent to $Y(\boldsymbol{x}, L, H)$. This allows us to rewrite the inner minimization problem (i.e., the wait-and-see problem) as follows, in terms of $\boldsymbol{x}, \boldsymbol{l}$ and $\boldsymbol{h}$.

$$\begin{aligned}
\text{minimize} \quad & \sum_{j\in V_2}\left(\sum_{i\in V_1} t_{ij}s_{ij} - p_j(\bar{d}_j - \tilde{d}_j l_j + \tilde{d}_j h_j)y_j\right) && && (7)\\
\text{s.t.} \quad & \sum_{i\in V_1} s_{ij} \ge y_j(\bar{d}_j - \tilde{d}_j l_j + \tilde{d}_j h_j) && \forall j \in V_2 && (8)\\
& (3), \boldsymbol{S} \in \mathbb{R}_+^{|V_1|\times|V_2|}, \boldsymbol{y} \in \{0,1\}^{|V_2|}
\end{aligned}$$

We start by linearizing every product arising between l_j, h_j and y_j within the constraints. This can be achieved by adding two continuous decisions variables z_j^l and z_j^h for all $j \in V_2$, subject to the following constraints.

$$z_j^l \le l_j \qquad z_j^l \le y_j \qquad z_j^l \ge y_j + l_j - 1 \qquad z_j^h \le h_j \qquad z_j^h \le y_j \qquad z_j^h \ge y_j + h_j - 1 \tag{9}$$

Let us introduce, for all $\boldsymbol{x} \in X$ and all $(\boldsymbol{h}, \boldsymbol{l}) \in \Xi$, set $Z(\boldsymbol{x}, \boldsymbol{l}, \boldsymbol{h})$ as the set of decision variables $(\boldsymbol{y}, \boldsymbol{S}, \boldsymbol{z}^l, \boldsymbol{z}^h)$ such that $\boldsymbol{y} \in \{0,1\}^{|V_2|}$, $\boldsymbol{S} \in \mathbb{R}_+^{|V_1|\times|V_2|}$, $\boldsymbol{z}^l \in \mathbb{R}_+^{|V_2|}$ and $\boldsymbol{z}^h \in \mathbb{R}_+^{|V_2|}$ and fulfilling capacity constraints (3), linearization constraints (9) as well as constraints (10) defined as (8) where bilinear terms have been substituted.

$$\sum_{i\in V_1} s_{ij} \ge \bar{d}_j y_j - \tilde{d}_j z_j^l + \tilde{d}_j z_j^h \quad j \in V_2 \tag{10}$$

In turn, it is clear that the wait-and-see problem is equivalent to the following problem.

$$\min_{(\boldsymbol{y},\boldsymbol{S},\boldsymbol{z}^l,\boldsymbol{z}^h)\in Z(\boldsymbol{x},\boldsymbol{l},\boldsymbol{h})} \left\{ \sum_{j\in V_2}\left(\sum_{i\in V_1} t_{ij}s_{ij} - p_j(\bar{d}_j - \tilde{d}_j l_j + \tilde{d}_j h_j)y_j\right)\right\} \tag{11}$$

Remark 2. *Constraints "$z_j^h \leq h_j$" and "$y_j - z_j^l \leq 1 - l_j$" can be omitted from $Z()$, by optimality.*

Proof. Note that variables $\boldsymbol{z}^h$ and $\boldsymbol{z}^l$ do not appear in the objective. Let $(\boldsymbol{l}, \boldsymbol{h}) \in \Xi$ be fixed.

- "$z_j^h \leq h_j$" is violated iff $z_j^h = 1$ and $h_j = 0$. Thus, assume that an optimal solution, having omitted the considered constraints, is such that $z_j^h = 1$ and $h_j = 0$. Then, the same solution with $z_j^h = 0$ is also feasible and fulfills the omitted constraints (since $\tilde{d}_j \geq 0$). Thus there exists a feasible solution with at most the same cost.
- Again, "$y_j - z_j^l \leq 1 - l_j$" is violated iff $z_j^l \neq y_j$ and $l_j = 1$. From $z_j^l \leq y_j$ we have that $y_j = 1$ and $z_j^l = 0$. Thus, assume that an optimal solution is such that $y_j = 1$ and $z_j^l = 0$. Then, the same solution with $z_j^l = 1$ is feasible and fulfills the omitted constraints.

□

In the next theorem, we reformulate the wait-and-see problem based on a polyhedral analysis result and Lagrangian duality. This theorem is the key ingredient for our reformulation.

Theorem 1. *For all $\boldsymbol{x} \in X$ and $(\boldsymbol{l}, \boldsymbol{h}) \in \Xi$, the wait-and-see problem is equivalent the following problem,*

$$\max_{(\boldsymbol{\lambda}^l, \boldsymbol{\lambda}^h) \geq 0} g(\boldsymbol{\lambda}^l, \boldsymbol{\lambda}^h, \boldsymbol{x}, \boldsymbol{l}, \boldsymbol{h}) \tag{12}$$

where $g(\boldsymbol{\lambda}^l, \boldsymbol{\lambda}^h; \boldsymbol{x}, \boldsymbol{l}, \boldsymbol{h})$ is defined as the optimal objective value of the following problem,

$$\min_{(\boldsymbol{y}, \boldsymbol{S}, \boldsymbol{z}^l, \boldsymbol{z}^h) \in Z_X(\boldsymbol{x})} \left\{ \sum_{j \in V_2} \left(\sum_{i \in V_1} t_{ij} s_{ij} - p_j(\bar{d}_j - \tilde{d}_j l_j + \tilde{d}_j h_j) y_j + \lambda_j^h h_j (y_j - z_j^h) + (1 - l_j) \lambda_j^l z_j^l \right) \right\} \tag{13}$$

where $Z_X()$ is defined as $Z()$ where constraints "$z_j^h \leq h_j$", "$z_j^l \geq y_j + l_j - 1$", "$y_j - z_j^l \leq 1 - l_j$" and "$z_j^h \geq y_j + h_j - 1$" have been omitted.

Proof. From Remark 2, one can omit constraints "$z_j^h \leq h_j$" and "$y_j - z_j^l \leq 1 - l_j$". We let $Z'()$ denote set $Z()$ where those constraints have been removed. Then, by linearity of the objective function in the wait-and-see problem, one can replace the feasible space by its convex hull. Noticing that, for all $(\boldsymbol{l}, \boldsymbol{h}) \in \Xi$, the following holds,

$$\text{conv}(Z'(\boldsymbol{x}, \boldsymbol{l}, \boldsymbol{h})) = \text{conv}(Z_X(\boldsymbol{x})) \cap \{(\boldsymbol{y}, \boldsymbol{S}, \boldsymbol{z}^l, \boldsymbol{z}^h) : z_j^l \leq l_j \text{ and } y_j - z_j^h \leq 1 - h_j\} \tag{14}$$

and using a Dantzig-Wolfe reformulation of $\text{conv}(Z_X(\boldsymbol{x}))$ for a fixed $\boldsymbol{x} \in X$, one can view the wait-and-see problem as an LP for which strong (partial) duality holds. This (partial) dual consists in maximizing function $\tilde{g}$, defined as follows, over $(\boldsymbol{\lambda}^l, \boldsymbol{\lambda}^h) \geq 0$.

$$\tilde{g}(\boldsymbol{\lambda}^l, \boldsymbol{\lambda}^h; \boldsymbol{x}, \boldsymbol{l}, \boldsymbol{h}) = \min_{(\boldsymbol{y}, \boldsymbol{S}, \boldsymbol{z}^l, \boldsymbol{z}^h) \in Z_X(\boldsymbol{x})} \left\{ \sum_{j \in V_2} \left(\sum_{i \in V_1} t_{ij} s_{ij} - p_j(\bar{d}_j - \tilde{d}_j l_j + \tilde{d}_j h_j) y_j \right) \right.$$
$$\left. + \sum_{j : l_j = 0} \lambda_j^l z_j^l + \sum_{j : h_j = 0} \lambda_j^h (y_j - 1 - z_j^h) + \sum_{j : l_j = 1} \lambda_j^l (z_j - 1) + \sum_{j : h_j = 1} \lambda_j^h (y_j - z_j^h) \right\} \tag{15}$$

By inspection, for all j such that $l_j = 1$ (resp. $h_j = 0$), $\lambda_j^l = 0$ (resp. $\lambda_j^h = 0$) is optimal. □

Substituting the wait-and-see problem from problem (4) with the reformulation introduced in Theorem 1, one obtains a conceptually simpler problem in which uncertainty interferes within the objective function only. However, the resulting problem contains bilinear terms in the objective. In the next corollary, we equivalently replace bilinear terms with fixed penalizations.

Corollary 1. *Let $\boldsymbol{\lambda}^{l*}(\boldsymbol{l})$ and $\boldsymbol{\lambda}^{h*}(\boldsymbol{h})$ be optimal for (12) for a given $(\boldsymbol{l}, \boldsymbol{h}) \in \Xi$ and let $\underline{\boldsymbol{\lambda}}^l$ (resp. $\underline{\boldsymbol{\lambda}}^h$) be such that $\underline{\lambda}_j^l \geq \lambda_j^{l*}(l_j) \geq 0$ (resp. $\underline{\lambda}_j^h \geq \lambda_j^{h*}(h_j) \geq 0$) for all $(\boldsymbol{l}, \boldsymbol{h}) \in \Xi$. Then,*

$$\max_{(\boldsymbol{\lambda}^l, \boldsymbol{\lambda}^h) \geq 0} g(\boldsymbol{\lambda}^l, \boldsymbol{\lambda}^h, \boldsymbol{x}, \boldsymbol{l}, \boldsymbol{h}) = g(\underline{\boldsymbol{\lambda}}^l, \underline{\boldsymbol{\lambda}}^h; \boldsymbol{x}, \boldsymbol{l}, \boldsymbol{h}) \tag{16}$$

Proof. By definition of $\boldsymbol{\lambda}^l(\boldsymbol{l})$ and $\boldsymbol{\lambda}^h(\boldsymbol{h})$, the dual (12) is equivalent to,

$$\min_{(\boldsymbol{y},\boldsymbol{S},\boldsymbol{z}^l,\boldsymbol{z}^h)\in Z_X(\boldsymbol{x})} \left\{ \sum_{j\in V_2} \left(\sum_{i\in V_1} t_{ij}s_{ij} - p_j(\bar{d}_j - \tilde{d}_j l_j + \tilde{d}_j h_j)y_j + \lambda_j^{h*}(\boldsymbol{h})h_j(y_j - z_j^h) + (1-l_j)\lambda_j^{l*}(\boldsymbol{l})z_j^l \right) \right\} \quad (17)$$

By optimality, since $\boldsymbol{\lambda}^{h*}(\boldsymbol{h}) \geq 0$ and $\boldsymbol{z}^h \leq \boldsymbol{y}$, any value greater than $\boldsymbol{\lambda}^{h*}(\boldsymbol{h})$ is also optimal for $\boldsymbol{\lambda}^h$. Similarly, since $\boldsymbol{z}^l \geq 0$, any value greater than $\boldsymbol{\lambda}^{l*}(\boldsymbol{l})$ is also optimal for $\boldsymbol{\lambda}^l$. □

Corollary 1 therefore eliminates the need of introducing dual variables, and thus, eliminates the bilinear terms from the objective. In the next theorem, we give valid values for $\underline{\boldsymbol{\lambda}}^l$ and $\underline{\boldsymbol{\lambda}}^h$.

Theorem 2. *In corollary 1, one can safely use $\underline{\boldsymbol{\lambda}}^l$ and $\underline{\boldsymbol{\lambda}}^h$ such that $\underline{\lambda}_j^l = p_j(\bar{d}_j - \tilde{d}_j)$ and $\underline{\lambda}_j^h = p_j(\bar{d}_j + \tilde{d}_j)$ for all $j \in V_2$.*

Proof. The dual (12) can be seen as maximizing a real variable θ subject to the following constraints with $(\boldsymbol{l}, \boldsymbol{h}) \in \Xi$: for all $(\boldsymbol{y}, \boldsymbol{S}, \boldsymbol{z}^l, \boldsymbol{z}^h) \in Z_X(\boldsymbol{x})$,

$$\theta \leq \sum_{j\in V_2} \left(\sum_{i\in V_1} t_{ij}s_{ij} - p_j(\bar{d}_j - \tilde{d}_j l_j + \tilde{d}_j h_j)y_j + \lambda_j^h h_j(y_j - z_j^h) + (1-l_j)\lambda_j^l z_j^l \right) \quad (18)$$

Firstly, note that it is always feasible not to serve any client if the transportation costs are higher than the profit. Moreover, note that $-\lambda_j^h h_j z_j^h \leq 0$. Thus, the following holds.

$$\theta - \sum_{i\in V_1}\sum_{j\in V_2} t_{ij}s_{ij} \leq \sum_{j\in V_2} \left(-p_j(\bar{d}_j - \tilde{d}_j l_j + \tilde{d}_j h_j)y_j + \lambda_j^h h_j y_j + (1-l_j)\lambda_j^l z_j^l \right) \leq 0 \quad (19)$$

This implies that it is enough to consider $(\boldsymbol{\lambda}^l, \boldsymbol{\lambda}^h)$-values such that the second part of the inequality holds. Clearly, the bounds provided by the theorem are enough. □

Theorem 2 achieves the ultimate goal of reformulating problem (4) as a two-stage robust problem with objective uncertainty. Introducing function Π as the objective function of the reformulated wait-and-see problem, see (21), problem (4) is equivalent to the following problem.

$$\min_{\boldsymbol{x}\in X} \max_{(\boldsymbol{l},\boldsymbol{h})\in\Xi} \min_{(\boldsymbol{y},\boldsymbol{S},\boldsymbol{z}^l,\boldsymbol{z}^h)\in Z_X(\boldsymbol{x})} \Pi(\boldsymbol{y}, \boldsymbol{S}, \boldsymbol{z}^l, \boldsymbol{z}^h, \boldsymbol{l}, \boldsymbol{h}) \quad (20)$$

$$\Pi(\boldsymbol{y}, \boldsymbol{S}, \boldsymbol{z}^l, \boldsymbol{z}^h, \boldsymbol{l}, \boldsymbol{h}) = \sum_{j\in V_2} \left(\sum_{i\in V_1} t_{ij}s_{ij} - p_j(\bar{d}_j - \tilde{d}_j l_j + \tilde{d}_j h_j)y_j + \underline{\lambda}_j^h h_j(y_j - z_j^h) + (1-l_j)\underline{\lambda}_j^l z_j^l \right) \quad (21)$$

3 Experimental results

In the previous section, we have reformulated problem (4) so as to obtain a cost-uncertain two-stage robust problem. This class of problems has been studied, among others, in [3] where the authors introduce a branch-and-cut algorithm for cases with binary here-and-now decisions. In this section, we apply their methodology to our reformulation of the two-stage robust FLP.

Instance generation. We generated instances according to [2]. For every site $i \in V_1$, the capacity q_i was uniformly generated between 10 and 160 while the opening cost was computed as $f_i = \alpha_i + \beta_i\sqrt{q_i}$ where α_i and β_i were generated between 0 and 90 and 100 and 110, respectively. The candidate positions for opening facilities and the location of clients were randomly generated in the unitary square. Then, for every connection $(i, j) \in V_1 \times V_2$, the transportation cost t_{ij} was defined as the associated Euclidean distance multiplied by 10. Demands were generated so that $\sum_{i\in V_1} q_i / \sum_{j\in V_2} \bar{d}_j = \gamma$ where γ is a parameter taking value 1.5 or 2, and $\tilde{d}_j$ was set to $\bar{d}_j$ multiplied by a randomly generated number between 0.00 and 0.25 (i.e., demands can vary up to 25%). Finally, every client's profit was set equal to $p_j = \frac{4}{|V_1|}\sum_{i\in V_1} t_{ij}$. Every input data was rounded to the closest integer. For each combination of $(|V_1|, |V_2|, \gamma)$, we generated 8 instances which were solved for $\Gamma \in \{2, 6, 8\}$.

$\|V_1\|$	$\|V_2\|$	Γ	$\gamma = 1.5$ # opt	time	$\gamma = 2.0$ # opt	time	Total # opt	time
6	12	2	8	0.2	8	0.3	16	0.3
		4	8	3.1	8	1.0	16	2.0
		6	8	4.2	8	1.1	16	2.6
8	16	2	8	1.6	8	0.6	16	1.1
		4	8	38.1	8	21.1	16	29.6
		6	8	139.7	8	64.4	16	102.0
10	20	2	8	6.9	8	2.2	16	4.5
		4	6	404.0	8	277.1	14	331.5
		6	2	1539.7	7	723.1	9	904.6
12	24	2	8	5.6	8	6.6	16	6.1
		4	7	1413.8	5	445.6	12	1010.4
		6	1	2314.2	3	1881.0	4	1989.3

TAB. 1: Computation time for different instance sizes of the robust FLP

Results analysis. Table 1 reports the outcome of our experiments. Column "opt" gives the number of instances (out of 8 for each value of γ) that are solved to proven optimality within the time limit, whereas "time" reports the average computing time, with respect to instances that are solved only. The results show that our reformulation is able to solve all instances of size $(6, 12)$ and $(8, 16)$. Larger instances seem to be harder to solve. However, we can see that our approach can solve every instance for small values of Γ (namely, $\Gamma = 2$) in less than ten seconds. Finally, we can notice that the instances with bigger values of γ are easier to solve in practice.

4 Conclusion

In this paper, we considered a facility location problem in which clients' demands are uncertain. We introduced a two-stage robust formulation of this problem with mixed-integer wait-and-see decisions and discrete uncertainty, a setting for which no viable exact approach has emerged in the literature. To tackle this problem, we have developed a non-trivial reformulation which turns our original problem into a two-stage robust problem in which the uncertainty only interferes in the objective function. This allowed us to rely on recent advances for cost uncertain adaptive robust problems to derive the first exact approach for this problem. We are currently working on extending this reformulation to a wider class of problems. From a computational viewpoint, we also reported early experimental results which showed that our approach is able to solve medium size instances to optimality.

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