Integrated cutting stock and batch replenishment problem

Ayse Akbalik[1], Elsa Silva[2], Maria Antónia Carravilla[3],
José Fernando Oliveira[3], Christophe Rapine[1]

[1] LCOMS, Université de Lorraine, Campus Metz, France
ayse.akbalik@univ-lorraine.fr, christophe.rapine@univ-lorraine.fr
[2] INESC TEC - Instituto de Engenharia de Sistemas e Computadores, Portugal
elsa.m.silva@inesctec.pt
[3] INESC TEC, Faculty of Engineering, University of Porto, Portugal
mac@fe.up.pt, jose.oliveira@fe.up.pt

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Problem statement and some results

The lot-sizing problem and the cutting stock problem have been the object of extensive research for more than 50 years. Much progress has been made concerning formulations and solution methods for these two problems. Most of the research has focused on solving these problems separately since each problem in itself can be difficult to solve. However, with the fast progress in optimization theory, software, hardware, and a better understanding of their structure and the dependencies among decisions observed in practical cases, more attention has been paid to the integration of these two problems in recent years. One motivation is to propose more environment-friendly replenishment planning together with cutting production planning. Note that the replenishment decisions directly impact the assortment of raw materials available in each period when solving the cutting stock problem. Integrated optimization of both problems aims to better optimize the shipping part while considering inventory holding and cutting costs and constraints, and better plan the cutting operations driven by the inbound shipping costs and available raw materials.

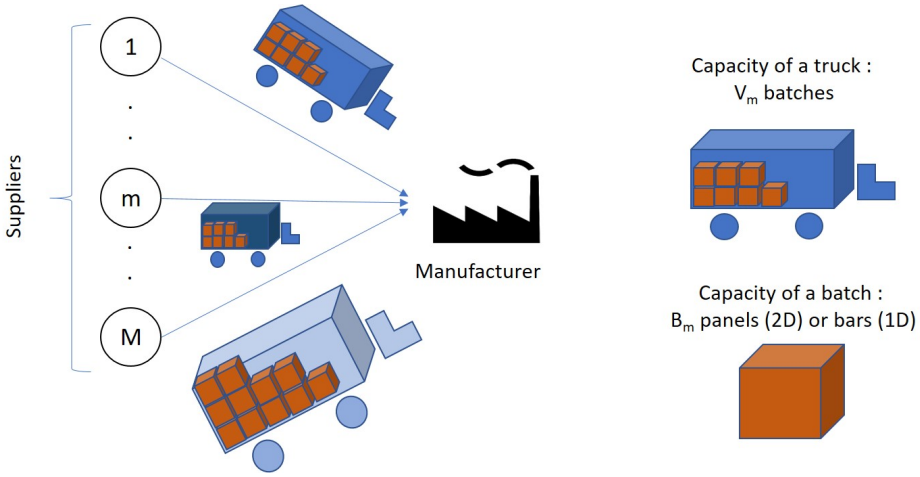

FIG. 1: Illustration of batch replenishment problem.

In this paper, the integrated Cutting Stock & Batch replenishment problem is studied. A manufacturer replenishes raw materials (rectangular panels in 2D or bars in 1D) from multiple suppliers using vehicles of a limited capacity. Replenishment of those panels/bars constitutes

the lot sizing part: how many capacitated vehicles to replenish in each period from each supplier loaded with batches containing panels/bars, while minimizing the total cost? The cutting process of those panels or bars into smaller pieces of different dimensions leads to the cutting stock problem: how to cut each panel (or bar) to satisfy the final demand of the customer for different types of end products while minimizing the total number of raw materials cut over the given time horizon? In our study, both problems are integrated into the same model to obtain the global optimal solution respecting replenishment cost and constraints together with cutting stock cost and constraints. For the lot sizing part, the costs consist of replenishment (fixed and unit costs) and inventory holding cost of those raw materials once arrived at the manufacturer site. Cutting stock costs include a fixed cutting cost per panel (or bar) cut and inventory holding cost of the final products in case of storage before their shipment to the customers.

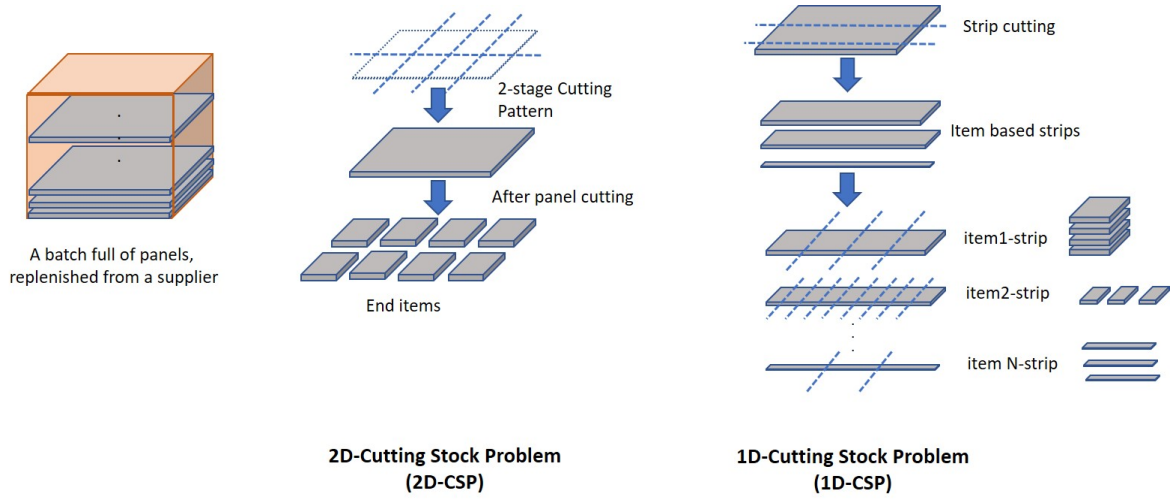

FIG. 2: Illustration of 1D and 2D Cutting Stock Problem.

To the best of our knowledge, no previous study had focused on batch replenishment coupled with cutting stock problem in the literature. The reader can refer to the recent review from Melega *et al.* [2] for more details on the integrated lot-sizing and cutting stock problems. We propose a mixed-integer linear programming (MILP) formulation for this integrated problem for both 2D and 1D cases, and propose some valid inequalities for 1D cutting stock & batch replenishment case. We also study some special cases of this integrated problem by decomposing it into subproblems to use some dynamic programming algorithms already existing in the literature. We have identified the following subproblem, which is polynomially solvable: one-dimensional multi-item, multi-period cutting stock problem without replenishment part. Very surprisingly, we have shown that a result from the lot sizing literature can be used to solve this pure cutting stock problem, in $O(T^3 log(T))$ time, see Li *et al.* [1].

References

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