A new branch-cut-and-price algorithm for the split delivery vehicle routing with time windows

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Keywords : *Branch-cut-and-price, split delivery vehicle routing with time windows.*

1 Introduction

The split delivery vehicle routing problem with time windows (SDVRPTW) derives from the basic capacitated vehicle routing problem (CVRP) with the addition of service time windows and the possibility of splitting deliveries. One can reckon adding such operational flexibility may allow better utilisation of the vehicles' capacity, which turns into routing savings that can reach up to 50%. However, this relevant margin comes in exchange for an increased difficulty in modeling and solving split delivery variants, especially when it comes to exact approaches.

Branch-cut-and-price (BCP) algorithms for the SDVRPTW can be found in [2] and [1]. More recently, branch-and-cut (BC) algorithms became the state-of-the-art approach (see [4]). Such algorithms can effectively solve many benchmark instances with up to 50 customers, but they become inefficient if the number of customers increases to 100. We propose a new BCP algorithm based on a novel property that allows us to determine the minimal delivery quantities to customers. Moreover, our BCP algorithm resorts to sophisticated techniques from the literature as well as known and new families of valid inequalities.

2 Problem definition and properties

We define the SDVRPTW over a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ on which the set of vertices is $\mathcal{V} = \{0, n+1\} \cup \mathcal{N}$, nodes 0 and $n+1$ standing for depot nodes and $\mathcal{N} = \{1, 2, \ldots, n\}$ standing for customer ones. The set of arcs $\mathcal{A} = \{(i, j) : i, j \in \mathcal{V},\ i \neq n+1,\ j \neq 0,\ i \neq j\}$ has non-negative costs c_{ij} and travel times t_{ij}, the triangle inequality holding for both. Each customer $i \in \mathcal{N}$ has an integer demand d_i and is available for service within the interval $[e_i, l_i]$, e_i and l_i standing for earliest and latest time to start the service, respectively. Waiting for starting the service is allowed. A fleet of vehicles with capacity Q is at our disposal. An SDVRPTW feasible solution is formed by a set of time and capacity feasible routes covering all customer demands.

It is well known from the problem's literature that there exists an optimal solution for which the following properties are known to hold: **1)** Two routes share at most one single split customer; **2)** Each route is elementary; **3)** Each arc between customer nodes is traversed at most once; **4)** For each pair of reverse arcs between customers, at most one is traversed; **5)** Delivery quantities are integer. We introduce the following novel property:

Property 6) There exists an optimal solution in which all delivery quantities to customers are multiples of the greatest common divisor of the capacity and all customer demands $m = gcd(Q, d_1, ..., d_n)$. The proof of this property is skipped due to the space restrictions.

3 Solution approach

We formulate the problem using one integer decision variable for each time and capacity feasible route and one constraint for each customer, which ensures that its demand is covered. We solve this model using a BCP algorithm. The pricing problem is the resource-constrained shortest path problem (RCSPP) with capacity and time resources. The RCSPP is defined on a multi-graph $\mathcal{G}' = (\mathcal{V}, \mathcal{A}')$ in which every arc $(i, j) \in \mathcal{A}$ is replaced with several arcs $(i, j) \in \mathcal{A}'$, each one corresponding to different possible delivery quantity. We take advantage of the property introduced in Section 2 to reduce the number of arcs in $\mathcal{A}'$. To solve the RCSPP we use the bucket-graph labeling algorithm introduced in [6].

To strengthen the formulation, we separate robust rounded capacity cuts. We also separate the following non-robust cuts: subset-row packing cuts adapted from the CVRP [3], subset-row covering cuts, and strong k-path cuts [1]. Non-robust cuts have a large impact on the pricing. Therefore, we use limited-memory technique [5] to reduce this impact. This is the first time in which limited-memory technique is adapted to subset-row covering and strong k-path cuts.

Our BCP algorithm also takes advantage of ng-path relaxation, arc elimination by reduced costs and multi-phase strong branching. We first branch on edges of graph $\mathcal{G}$. If all edge values are integer, Ryan&Foster branching is used, as in [2].

4 Preliminary results

We performed our computational tests with single cores having a Cascade Lake Intel® Xeon® Skylake Gold 6240 configuration with 2,6GHz processing frequency and 5,3GB RAM memory per core. We tested our algorithm on all instances of Solomon's VRPTW benchmark set allowing split deliveries and using vehicle capacities 30, 50 and 100.

We initialize the BCP algorithm with the primal bound obtained with a metaheuristic algorithm executed during 30, 60 and 120 seconds for instances with 25, 50 and 100 customers, respectively. The overall time limit is set to 1 hour per instance.

We were able to solve to optimality 153 from 168 instances with 50 customers, and 53 from 168 with 100 customers, whereas the best performing approaches in the literature are [4] who solved 120 instances with 50 customers, and [1] who solved 8 instances with 100 customers.

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