Tabu Search Exploiting Local Optimality in Binary Optimization

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Tabu search (TS) is a metaheuristic introduced by Glover (1986) that guides a local search heuristic to explore the solution space beyond local optimality. Tabu search incorporates adaptive memory and responsive exploration (see e.g. Glover and Laguna (1997), Glover and Hanafi (2002)). In this conference, we will present a new extension of TS called the Alternating Ascent (AA) algorithm in Glover (2020). The AA Algorithm incorporates strategies specially designed to exploit local optimality within the context of binary combinatorial optimization. In outline, the AA Algorithm alternates between an Ascent Phase and a Post-Ascent Phase using thresholds to identify variables to change their values and to transition from one phase to another. A high-level overview of the AA Algorithm (that removes essential features subsequently described) is as follows:

Overview of an Alternating Ascent (AA) Algorithm

While an outer loop termination criterion is not met **do**
    Choose a starting solution
    While an inner loop termination criterion is not met **do**
        Execute the following two phases:
        Ascent Phase: go to a local optimum
            (which may also be the starting solution on the first pass)
        Post Ascent Phase: move away from the local optimum *and*
            away from some number of other previous local optima.
    Endwhile
Endwhile

In outline, the AA Algorithm alternates between an Ascent Phase and a Post-Ascent Phase using thresholds to identify variables to change their values and to transition from one phase to another. The thresholds embody a form of adaptive memory based on a function called exponential extrapolation, which makes it possible to track the number of times that variables receive their current values in any selected number Q of most recent local optima represented by the set $\mathbb{Q}$. An exponential extrapolation measure $EE_j(\mathbb{Q}, x)$ is associated with a variable x_j that gives rise to a *recency threshold* of the form $EE_j(\mathbb{Q}, x) \geq Threshold_r(\mathbb{Q})$, which assures that changing the current value for x_j will not duplicate its value in the r most recent local optima. By reference to a standard evaluation $Eval_j(x)$ for x_j that identifies the change in the objective function when x_j changes its value, and taking advantage of a rudimentary tabu search restriction and aspiration criterion, this in turn gives rise to two status conditions denoted by $S^{=}$ and $S^{\neq}$, where an $S^{=}$ status identifies a variable that should change the value it received in the most recent local optimum and an $S^{\neq}$ status identifies a variable that should retain its value that differs from its value received in the most recent local optimum. These conditions are additionally exploited using counters $nS^{=}$ and $nS^{\neq}$ of the number of variables that have an $S^{\neq}$ and $S^{=}$ status, embodied in a *trigger threshold* of the form $nS^{=} + nS^{\neq} \geq Trigger$. The trigger threshold determines when a new Ascent Phase should be launched by removing all tabu restrictions except the one that caused the threshold to be satisfied. The resulting ascent first reaches a conditional local optimum where the last tabu restriction remains in force, and where it is assured that the solution cannot duplicate any of the r most recent local optima. Then this last restriction is also removed to complete the ascent to a true local optimum, and to begin a new Post-Ascent Phase.

Once no more improving moves remain (for the non-tabu variables) in an Ascent Phase, the resulting ascent reaches a *conditional local optimum* (subject to keeping x_k at its new value). At this point, we may remove the tabu restriction on x_k as well, to continue to a solution that is a true local optimum which ends the Ascent Phase. Given that the conditional local optimum does not duplicate the previous local optimum, and that the choice of moves leading to this conditional local optimum is influenced by the value assigned to x_k, there is a strong likelihood that the new local optimum will also differ from the previous local optimum.

To exploit this observation, we have to decide of whether to immediately use the change from $Eval_j(x) \leq 0$ to $Eval_j(x) > 0$ to trigger an ascent to a conditional local optimum, or whether to wait until more than one variable x_j selected to be x_k has undergone this change before launching such an ascent.

This study introduces a general procedure for launching a new ascent based on exponential extrapolation to exploit local optimality without recording the local optima. Exponential extrapolation provides a significant saving of both memory and computation over consulting the actual values of variables in previous local optima. Numerical examples are given to illustrate the use of exponential extrapolation and the key processes involved in exploiting local optimality via the recency and trigger thresholds. The present paper focuses on the simplest version of the TS metaheuristic exploiting local optimality in binary optimization, without including intensification, diversification, path relinking or multi pass strategies.

References

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