A Non-cooperative Game for the Freshness of Status Updates

Fernando Miguelez[1], Josu Doncel[2], Urtzi Ayesta[3,4,5,6] *

[1] Universidad Pública de Navarra, Pamplona, Spain
erlantz.lxt@gmail.com
[2] University of the Basque Country, UPV/EHU, Leioa, Spain
josu.doncel@ehu.eus
[3] UPV/EHU, University of the Basque Country, 20018 Donostia, Spain
[4] Ikerbasque- Basque Foundation for Science, 48009 Bilbao, Spain
[5] CNRS, IRIT,31071 Toulouse, France
[6] Université de Toulouse, INP, 31071 Toulouse, France
urtzi.ayesta@irit.fr

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1 Introduction

Timely information is a crucial factor in a wide range of information, communication, and control systems. For instance, in autonomous driving systems, the state of the traffic and the location of the vehicles must be as recent as possible. The Age of Information is a relatively new metric that measures the freshness of the knowledge we have about the status of a remote system. More specifically, the Age of Information is the time elapsed since the generation of the last successfully received packet by the monitor containing information about the source. Since the seminal paper [1], in several models it has been observed that the policies that optimize performance metrics of interest in queueing theory do not necessarily minimize the Age of Information. Hence, there is a large number of queueing models that are open research problems regarding the Age of Information metric. We refer to [3] for a recent survey of the Age of Information.

2 Game Formulation

We consider a system formed by K sources of information whose status is observed by a remote monitor. Each source generates status updates and sends them immediately through a transmission channel to the monitor. We assume that the transmission times from the sources to the transmission channel and from the transmission channel to the monitor are both negligible. Thus, the generation time of updates and the time at which updates arrive to the transmission channel coincide and similarly, the time at which updates end service and the delivery time of updates to the monitor coincide. The transmission channel we consider is a Last-Generated-First-Served queue with preemption in service (LGFS-PR). Hence, when an update arrives to the queue it starts being served immediately, preempting the update currently in service if any. The updates of source i are generated according to a Poisson process of rate λ_i. We assume that the service time of updates of source i is exponentially distributed with rate μ_i. Let $\vec{\mu} = (\mu_1, \dots, \mu_K)$, $\vec{\lambda} = (\lambda_1, \dots, \lambda_K)$, and $\lambda = \sum_i \lambda_i$. The Age of Information of source i at time t is defined as the difference between t and the generation time of the last update of source i that has been delivered to the monitor. We denote by $\Delta_i(\vec{\lambda}, \vec{\mu})$ the Average Age of Information (AAoI) of source i. We consider that the system planner penalizes a source if it

*This is a short version of the article that appeared in [2] generates updates very frequently. Specifically, we assume that each source must pay a cost for sending status updates that is proportional to its load, i.e., when the source i sends updates with rate λ_i, the payment of source i is $c\lambda_i/\mu_i$, where $c > 0$. The cost function of source i is defined as the sum of its AAoI and its payment , i.e.,

$$C_i(\vec{\lambda}, \vec{\mu}, c) = \Delta_i(\vec{\lambda}, \vec{\mu}) + c\lambda_i/\mu_i. \tag{COST}$$

We define a game where each source is a player and aims to choose its generation rate so as to minimize its cost function, i.e.,

$$\min_{\lambda_i} C_i(\vec{\lambda}, \vec{\mu}, c). \tag{GAME}$$

Let $\vec{\lambda}_{-i} = (\lambda_1, \ldots, \lambda_{i-1}, \lambda_{i+1}, \ldots, \lambda_K)$ and $BR(\vec{\lambda}_{-i}, \vec{\mu}, c)$ be the best response of player i to $\vec{\lambda}_{-i}$, that is, the optimal generation rate of source i when the remaining sources have generation rates $\vec{\lambda}_{-i}$. A solution of (GAME) is called a *Nash Equilibrium* and we denote it as $\vec{\lambda}^{ne} = (\lambda_1^{ne}, \ldots, \lambda_K^{ne})$. It is defined as a set of generation rates such that no source gets benefit from a unilateral deviation, i.e., for $i = 1, \ldots, K$ $\lambda_i^{ne} \in BR(\vec{\lambda}_{-i}^{ne}, \vec{\mu}, c)$.

We now focus on the global optimization problem of this model. It consists of finding a set of generation rates such that the cost functions aggregated across sources is minimized, i.e.,

$$\min_{\{\lambda_1, \ldots, \lambda_K\}} \sum_{i=1}^{K} C_i(\vec{\lambda}, \vec{\mu}, c). \tag{GLOBAL-OPT}$$

A solution of (GLOBAL-OPT) is denoted by $\vec{\lambda}^G$.

The Price of Anarchy (PoA) is a widely studied performance metric to analyse the inefficiency of the Nash equilibria, and is defined as the worst possible ratio between the cost at Nash Equilibrium and that of the global optimum. For this model, it is given by

$$PoA = \sup_{\vec{\mu}, c} \frac{\sum_{i=1}^{K} C_i(\vec{\lambda}^{ne}, \vec{\mu}, c)}{\sum_{i=1}^{K} C_i(\vec{\lambda}^{G}, \vec{\mu}, c)}. \tag{PoA}$$

3 Main Results

We aim to characterize the PoA. We first focus on a system with homogeneous service rates, i.e., when $\mu_i = \mu$ and then when the service rates are heterogeneous.

Proposition 1 *In a system with homogeneous service rates, the PoA is* $2 - \frac{1}{K}$.

Proposition 2 *In a system with heterogeneous service rates, the PoA is unbounded from above.*

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Références

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