Qualifying quantum approaches for hard industrial optimization problems. A case study in the field of smart-charging of electric vehicles

Constantin Dalyac[1,2], Loïc Henriet*[1], Emmanuel Jeandel[3], Wolfgang Lechner[4,5], Simon Perdrix[3], Marc Porcheron[6], and Margarita Veshchezerova[3,6]

[1]Pasqal, 2 avenue Augustin Fresnel, 91120 Palaiseau, France
[2]LIP6, CNRS, Sorbonne Université, 4 Place Jussieu, 75005 Paris, France
[3]LORIA, CNRS, Université de Lorraine, Inria Mocqua, F 54000 Nancy, France
[4]University of Innsbruck, Technikerstrasse 21a, Innsbruck, Austria
[5]Parity Quantum Computing GmbH, Rennweg 1, Innsbruck, Austria
[6]EDF R&D, 7, boulevard Gaspard Monge, 91120 Palaiseau, France

1 Introduction

In the quest for a quantum advantage, situating quantum algorithms and technologies with respect to classical approaches for solving NP-hard problems is a key question. Seminal works from Shor and Grover were a major breakthrough in this perspective in the early developments of quantum computing. Based on the theoretical model of quantum computation, they developed quantum algorithms that provide respectively an exponential speed-up for integer factorization [1], and a quadratic speed-up on exhaustive search in an unstructured database [2], a principle applicable to any NP problem providing at most such a speed-up in terms of query complexity [3, 4]. However, implementing these algorithms requires fault-tolerant quantum machines handling a large number of qubits, which have yet to be built.

While many technological obstacles currently impede the creation of such machines, experimental physicists have been capable of controlling quantum systems precisely enough to simulate complex many-body quantum systems. These quantum devices present strong quantum properties and offer scientists a control on the quantum aspects of physical systems. They can have sizes of several hundreds of quantum particles, and because of the unavoidable coupling between the system and its environment, these quantum platforms fall in the category of Noisy Intermediate Scale Quantum (NISQ) devices [5]. Among them, there is a strong belief that Analog Quantum Simulators (AQS) can perform specific tasks intractable for classical computers in polynomial time, such as the dynamical simulation of strongly interacting quantum Hamiltonians [6, 7], and it is expected that AQS will be among the first to propose useful applications in the short-term [8]. Lately, there has been growing interest in knowing if the quantum characteristics of these devices can be steered towards outperforming classical computers on industry-relevant tasks. An active field of research is currently guided towards combinatorial optimization, where the Hilbert space spanned by the many-body quantum system is used to efficiently encode a high-dimensional discrete problem.

In this context, algorithms that can run on such NISQ devices have been developed. Quantum Annealing [9], the Quantum Adiabatic Algorithm [10], and the Quantum Approximate Optimization

*corresponding author : loic@pasqal.io

Algorithm [11] are among the most promising ones. A better understanding of the performances of these approaches on industrial NP-hard problems is of great interest, both for quantum computing adoption and for the application domains concerned.

In this article, we present and discuss a case study based on QAOA for two problems drawn from the rapidly growing sector of smart-charging of electrical vehicles in which EDF, the French utility for electricity production and supply, is strongly involved. Once appropriately modeled, these problems appear as classical NP-hard graph theory problems, Max-k-Cut and Maximum Independent Set (MIS) respectively. For the problems under consideration, we provide some implementation details on such platform as developed and commercialized by the company Pasqal.

2 Two smart-charging problems and their modeling as NP-hard problems

Both problems will be tackled under the following assumptions:

- A load station is made up of several charging points, each of them loading at most a single electric vehicle (EV) at a given time step;
- The charging points are *parallel identical* machines that supply the same power. The charging time of a given EV is thus independent of the charging point it is scheduled on;
- We consider neither additional job characteristics and constraints (release/due dates, charging profile imposed by the battery state) nor global resource constraints on the load station (maximal power deliverable at a given time step);
- Preemption is not allowed: a load task cannot be interrupted to be resumed later, on the same charging point or another one.

2.1 Minimization of Total Weighted Load Completion Time (SC1) and Max-k-Cut

We consider $J = \{1, \ldots, n\}$ charging jobs of n EVs with durations $T = \{t_1, \ldots, t_n\}$, to be scheduled on a set $I = \{1, \ldots, k\}$ of k charging points. An integer weight $w_j > 0$ is associated to each job j, measuring its importance. For example, we might want to prioritize the charge of emergency vehicles. The time at which a load j ends, called the *completion time* is noted C_j and we want to minimize the weighted total time of completion of the charges $\sum_{j \in J} w_j C_j$.

(SC1) is a classical scheduling problem known to be NP-hard in the general case, polynomial on a single machine or without priorities/weights attached to the jobs [12]. If the number of machines m is fixed, (SC1) is NP-hard in the weak sense that it can be solved by pseudo-polynomial algorithms[1], typically based on dynamic programming. In the case where m is not fixed, (SC1) is NP-Hard in the strong sense, meaning that no such pseudo-polynomial algorithm exists except if P=NP.

We analyze their performances on some graphs drawn from real-world problems considering the performances obtained by the best-known randomized approximation classical algorithms. We observe that QAOA outperforms Goemans and Williamson ratio on these particular instances of Max-Cut. Similar results were observed in Ref. [13] on small random graphs. Establishing that such improvement would hold true for worst-case and large graphs instances is still an open question, and will remain so as long as we stay in the NISQ era. In any case, reaching similar approximation ratios as classical solutions,

[1] An algorithm is said to be *pseudo-polynomial* if it is polynomial in the numeric values of its data, but super-polynomial in the length of their binary encoding.

but faster or at a lower energy cost would already be of significant value for industries.

2.2 Optimal Scheduling of Load Time Intervals within Groups (SC2) and MIS

We now consider the following problem (SC2): given a set of load tasks represented as *intervals* on a timeline, such that each of them belongs to a specific *group*, for example distinct vehicle fleets of a company, select a subset of these loads (i) which maximizes the number of non-overlapping tasks and (ii) such that at most one load in each group is completed. The goal is here to both minimize the completion time of the selected loads and to guarantee that no group will be over-represented in the schedule.

This problem belongs to the class of *Interval Scheduling* problems [14]. More precisely, it is a *Group Interval Scheduling*, or *Job Interval Selection* problem. It can be restricted without loss of generality to the case where all the groups contain the same number of tasks, k. It is NP-Complete for $k \geq 3$, and has no PTAS for $k \geq 2$ unless $P = NP$ [15]. Some polynomial approximation ratios have been obtained in the general case, namely 0.5 in Ref. [15], improved to 0.63211 in Ref. [16], while polynomial algorithms exist in cases where some parameters are fixed [17].

3 Quantum approaches to smart-charging problems

For these two problems, we adapt the "Quantum Approximate Optimization Algorithm" (QAOA) that computes approximate solutions to combinatorial optimization problems, with a theoretical guarantee of convergence when the depth of the quantum circuit increases [11].

QAOA is a variational algorithm for combinatorial problems in which a quantum processor works hand-in-hand with a classical counterpart, as illustrated in Fig. 1.

In our paper, we will consider two distinct ways to prepare the ansatz wavefunction on Pasqal quantum processors. In the first one, the quantum processor is used in digital mode, and the trial wavefunction is the output of a quantum circuit composed of discrete quantum gates. In the second one, the quantum processor is used in an analog manner and the trial wavefunction results from the application of a continuously parameterized Hamiltonian.

Detailed numerical simulations can be found in the full-version of this paper.

4 Conclusions and Perspectives

Qualifying quantum algorithms on difficult optimization problems is of great importance to evaluate the benefit of quantum computing, as these problems are at the core of many industrial applications where they often constitute performance bottlenecks.

Two major principles must be implemented in such a process:

1. Rely on a collaboration between experts in the application field under study and quantum computing experts, in order to design fine-tuned, *ad-hoc*, software-hardware solutions;

2. Benchmark quantum algorithms not only against exact classical algorithms, by nature exponential on this class of problems, but also versus available approximate polynomial ones.

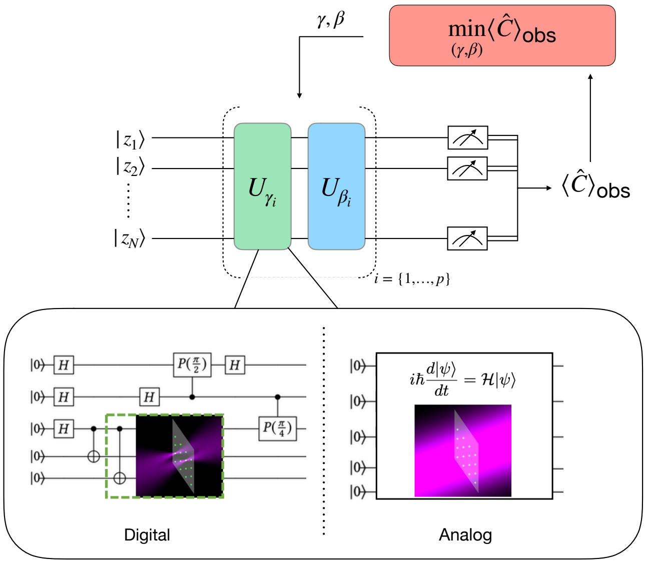

FIG. 1: Principle of the QAOA algorithm. A quantum processor, which can be operated in either digital or analog mode, is used to prepare an ansatz wavefunction from which we construct the mean value $\langle \hat{C} \rangle_{\text{obs}}$ using numerous measurements. A classical optimizer then updates the variational parameters. Some problems are naturally tailored to the analog mode of the platform, while others require a digital mapping. In digital mode, the unitaries are built from quantum circuits, made of elementary quantum gates acting each on one or a few qubits. In the analog mode, the unitaries are built from sequences of Hamiltonians that can be controlled in a continuous manner.

This paper reports a case study based on this protocol in the field of smart-charging of electric vehicles. We specified two smart-charging problems, which, although stylized to be treated by available quantum approaches, stay representative of the real operational problems currently solved by the EDF subsidiaries involved in the field. We developed a hardware-efficient implementation of QAOA on quantum devices based on Rydberg atoms arrays to solve these two problems, respectively modeled as "sub-difficult" instances of Max-k-Cut and MIS NP-hard problems. We implemented these procedures on a real data-set of 2250 loads, and compared quantum solutions to classical approximate ones, up to current limits of classical simulation of quantum hardware with $N \leq 20$ qubits. In both cases, quantum algorithms behave correctly, obtaining high approximation ratios, coherently with the fact that both applications are modeled as "less difficult" instances than the worst-case ones of these two NP-Complete problems.

These results, obtained through a rigorous protocol, are very encouraging. Future works will involve testing the quantum approaches on the real Rydberg atoms quantum processor developed by Pasqal in the 100-1000 qubits range [18]; making the smart-charging problems more realistic by incorporating new constraints (e.g maximal available power on the load station), a real challenge as this should make the associated Hamiltonians to be implemented on the processor more complex; more specifically from an application viewpoint, looking for efficient heuristics to transform general graphs in unit-disk ones, which would drastically simplify the procedure for quantum solving of MIS. On this latter point, another interesting option is to explore smart-charging problems which are "naturally" two dimensional – e.g. based on the "autonomy radius" of vehicles or the "action radius" of charging points –, thus replacing the costly and hypothetical resolution of the (UD) problem by a simple scale reduction in the plan.

References

1. Shor, P. W. *Algorithms for quantum computation: discrete logarithms and factoring* in *Proceedings 35th Annual Symposium on Foundations of Computer Science* (1994), 124–134.
2. Grover, L. K. *A Fast Quantum Mechanical Algorithm for Database Search* in *Proceedings of the Twenty-Eighth Annual ACM Symposium on Theory of Computing* (Association for Computing Machinery, Philadelphia, Pennsylvania, USA, 1996), 212–219. ISBN: 0897917855. https://doi.org/10.1145/237814.237866.
3. Bennett, C. H., Bernstein, E., Brassard, G. & Vazirani, U. Strengths and Weaknesses of Quantum Computing. *SIAM Journal on Computing* **26,** 1510–1523. ISSN: 1095-7111. http://dx.doi.org/10.1137/S0097539796300933 (Oct. 1997).
4. Viamontes, G. F., Markov, I. L. & Hayes, J. P. Is quantum search practical? *Computing in science & engineering* **7,** 62–70 (2005).
5. Preskill, J. Quantum Computing in the NISQ era and beyond. *Quantum* **2,** 79. ISSN: 2521-327X. http://dx.doi.org/10.22331/q-2018-08-06-79 (Aug. 2018).
6. Choi, J.-y. *et al.* Exploring the many-body localization transition in two dimensions. *Science* **352,** 1547–1552. ISSN: 1095-9203. http://dx.doi.org/10.1126/science.aaf8834 (June 2016).
7. Scholl, P. *et al.* Programmable quantum simulation of 2D antiferromagnets with hundreds of Rydberg atoms. *arXiv e-prints,* arXiv:2012.12268. arXiv: 2012.12268 [quant-ph] (Dec. 2020).
8. Kendon, V. Quantum computing using continuous-time evolution. *arXiv:2004.00704* (2020).
9. Satoshi, M. & Hidetoshi, N. Mathematical Foundation of Quantum Annealing. *arXiv:0806.1859* (2008).
10. Farhi, E., Goldstone, J., Gutmann, S. & Sipser, M. Quantum Computation by Adiabatic Evolution. *arXiv:quant-ph/0001106.* (2000).
11. Farhi, E., Goldstone, J. & Gutmann, S. A Quantum Approximate Optimization Algorithm. *arXiv:1411.4028* (2014).
12. Graham, R. L., Lawler, E., Lenstra, J. K. & Rinnooy Kan, A. H. G. Optimization and approximation in deterministic sequencing and scheduling: A survey. *Annals of Discrete Mathematics* **5,** 287–326 (1979).
13. Crooks, G. E. Performance of the Quantum Approximate Optimization Algorithm on the Maximum Cut Problem. *arXiv e-prints,* arXiv:1811.08419. arXiv: 1811.08419 [quant-ph] (Nov. 2018).
14. Kolen, A. W., Lenstra, J. K., Papadimitriou, C. H. & Spieksma, F. C. *Interval Scheduling: A Survey* www.interscience.wiley.com. Wiley InterScience, 2007.
15. Spieksma, F. C. R. On the approximability of an interval scheduling problem. *J. of Scheduling* **2(5),** 215–227 (1999).
16. Chuzhoy, J., Ostrovsky, R. & Y., R. Approximation algorithms for the Job interval selection problem and related scheduling problemsInterval. *Mathematics of Operations Research* **31(4),** 730–738 (2006).
17. Bevern, R., Mnich, M., Niedermeier, R. & Mathias, W. Interval Scheduling and Colorful Independent Sets. *arXiv:1402.0851v2* (2014).
18. Henriet, L. *et al.* Quantum computing with neutral atoms. *Quantum* **4,** 327. ISSN: 2521-327X. https://doi.org/10.22331/q-2020-09-21-327 (Sept. 2020).