A new algorithm for Optimization a Linear Function over Integer Efficient Set

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1 Introduction

In many important decision problems, a decisions-makers bases their choices on several objectives that must be optimized simultaneously, they see a need to examine solutions with multiple criteria which leads us to research in the domain of multi-objective optimization even if it is linear, integer, or non linear.

In our case, we concentrate on Multiobjective integer linear programs (MOILPs) which is a set of solutions that define the best compromise between competing objectives called set of nondominated points, often its cardinality is very large, it becomes confusing for the decision maker to choose the best compromise that represents his preferences especially if he only interested in the one that optimizes a new criterion also the time to calculate all the efficient solutions increases exponentially, which leads us to solve a problem entitled Optimization over efficient set.

In the literature many algorithms are used for this problems in the discrete case :(Nguyen (1992), Jorge (2009), Abbas and Chaabane (2006), Chaabane and Pirlot (2010), Chaabane, Brahmi and Ramdani (2012), Boland, N., Charkhgard, H., Savelsbergh, M., (2017)). In this communication, we propose a new exact method to optimize a linear function over the integer efficient set. The work is inspired from the recent developed idea **Craig A. Piercy, Ralph E. Steuer 2019**. The method is articulated on three main parts : decomposition (subdividing the criteria cone and sub-cone), evaluation (the main objective) and separation (after reducing the admissible region).

2 Preliminary and definitions

Consider the Multiple Objective Integer Linear Programming (MO) problem

$$(MO)\left\{\begin{array}{lcl} \text{"OPTIMIZE"} f(x) & = & Cx \\ s.t. & & Ax \leq b \\ & & x \geq 0 \end{array}\right. \tag{1}$$

Where $C \in \mathbb{R}^{p\times n}$, $Cx = (f_1(x) f_2(x) \cdots f_p(x))$, $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^{m\times 1}$.

To solve 1, it is necessary to improve at best the various objectives at the same time, solving this last one leads to a multitude of solutions called efficient solutions, thus leads to define it :

Définition 1 *A feasible solution $x_{ef} \in D$ is said to be an efficient solution of (MO) if and only if there is no feasible solution $x \in D$ such that $Cx \geq Cx_{ef}$ and $Cx \neq Cx_{ef}$($c^i x \geq c^i x_{ef}$ for all $i = 1, \cdots, p$ and $c^i x > c^i x_{ef}$ for at least one i).*

We note by χ_E the set of all efficient solutions, the problem of optimizing a linear function over the whole integer efficient set is defined as :

$$(MP)\left\{\begin{array}{lcl} \text{"max"} F(x) & = & fx \\ & & x \in \chi_E \end{array}\right. \tag{2}$$

Where $F(x)$ is a linear function. To assure that the problem cannot be solved directly, we assume that $x \neq \chi_E$.

Définition 2 *Let* $\bar{x} \in S$ *and let* $C^{\geq}$ *be the semi positive polar cone generated by the gradient by the gradients of the k objective functions where :*
$C^{\geq} = \{y \in \mathbb{R}^n | Cy \geq 0, Cy \neq 0\} \cup \{0 \in \mathbb{R}^n\}$.
It is the cone which is defined by the normal vectors N1 and N2 of the criterion vectors c1 and c2 respectively.

Définition 3 *The domination set :* $D_{\bar{x}} = \{\bar{x}\} \oplus C^{\geq}$*, is given by the set addition of* $\bar{x}$ *and* $C^{\geq}$*, in other words :*
$D_{\bar{x}} = \{x \in \mathbb{R}^n | x = \bar{x} + y, Cy \geq 0, Cy \neq 0\}$

3 Decomposition of the criteria cone

The decomposition of the proposed criterion cone consists in dividing the cone generated by the criteria into two sub-cones whose union of their sets of efficient solutions is equal to the set of efficient solutions of the criterion cone.

4 Conclusion

In this paper, we propose an new exact method for optimizing a linear function over integer efficient set by proposing a decomposition of the criteria cone of Multiobjective Integer Linear Programming problem without missing any solution.

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