A Hybrid Strategy for Solving the Bi-Objective Obnoxious p-Median Problem

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Mots-clés : *Learning, Multi-objective, Obnoxious p-median, Optimization, Population.*

1 Introduction

The obnoxious p-median problem occurs in real-world applications, where facilities have obnoxious features. Often an obnoxious facility induces a dangerous influence regarding the surrounding area. The bi-objective version of the obnoxious problem (noted Bi-OpM) occurs in more complex situations related to location of hazardous facilities, like nuclear or chemical power plants, waste storage facilities, and noisy or polluting services. The Bi-OpM is characterized by a set I of clients, a set J of facilities. It consists in selecting a subset of p facilities from a given set of possible locations to optimizing two antagonistic objective functions: (i) to maximize the sum of the minimum distance between each customer and its nearest open facility dc_{ij_k} and, (ii) to maximize the dispersion between facilities $df_{j_{k1}j_{k2}}$, where $j_{k1}, j_{k2} \in S, S \subset J, j_{k1} < j_{k2}$. Facilities belonging to S denote the open facilities, while those contained in $J \backslash S$ represent the closed or non-open facilities. The formal description of the bi-objective optimization case may be stated as follows (cf., Colmenar *et al.* [2]):

$$\text{Bi-OpM}\begin{cases} \max f_1 = \sum_{i\in I} \min_{k\in S} \left\{dc_{i_k} : j_k \in S\right\} \\ \max f_2 = \sum_{j_{k1}\in S} \min_{j_{k2}\in S} \left\{df_{j_{k1}j_{k2}} : j_{k1} < j_{k2}\right\} \\ \text{s.t.} \\ \qquad S \subseteq J, \text{ and } |S| = p \end{cases}$$

Early work on the bi-objective obnoxious p-median (Bi-OpM) problem was addressed by Colmenar *et al.* [2], who designed a memetic algorithm to approximately solve it and, Sánchez-Oro *et al.* [3] tackled that version by using a parallel variable neighborhood search. Another memetic algorithm was proposed by Alamatsaz *et al.* [1] for tackling the multi-objective obnoxious waste location-routing problem.

2 A hybrid strategy for solving the bi-objective version

The proposed method is based upon the following hybridization. First, a starting reference archive set of diversified solutions (approximate Pareto front) is built by tailoring a special greedy procedure, where the scalarizing version of the problem is considered. The achieved set of solutions is updated by calling an augmented greedy procedure, where a special criterion is considered. Second, an intensification step is combined with the drop/rebuild operator for maintaining a high degree of diversity of the population. Each current solution is submitted to an enhancement according to either the first or the second objective function, and only non-dominated solutions are selected to complete the best current approximate set. Third and last, an ϵ-constraint approach, where a series of ϵ-constraints are added to the current problem, is called for iteratively highlighting the final approximate Pareto front.

3 Experimental part

In the quantitative study, the performance of the proposed Hybrid Strategy (noted HS) was evaluated on some instances from a recent benchmark of the $Bi-OpM$ [3] by considering the hypervolume indicator that measures the volume of the hyperdimensional space dominated by a set of points A (in the objectives space):

#Inst	$I_{NH}^{\text{Mo-PVNS}}$ [3]	I_{NH}^{AOLS} [3]	I_{NH}^{DBLS} [3]	$I_{NH}^{\text{NSGA-II}}$ [3]	I_{NH}^{SPEA2} [3]	$I_{NH}^{-\text{HS}}$
pmed17.p25	0.968	0.999	**1.000**	0.999	0.999	0.860
pmed20.p50	0.737	0.729	0.727	0.616	0.715	**1.000**
pmed22.p62	**1.000**	0.972	0.694	0.675	0.914	0.997
pmed28.p75	**1.000**	0.958	0.950	0.628	0.819	0.895
pmed33.p87	0.863	0.748	0.743	0.372	0.549	**1.000**
pmed36.p100	**1.000**	0.818	0.825	0.346	0.533	0.965
pmed39.p112	**1.000**	0.821	0.820	0.328	0.514	0.791
pmed40.p225	0.862	0.625	0.637	0.196	0.271	**1.000**
Average	0.929	0.834	0.799	0.520	0.665	**0.934**

TAB. 1: Variation of the normalized hypervolume value on eight instances of the literature.

Table 1 reports the normalized hypervolume. Whenever the hypervolume is calculated, each normalized value is provided by dividing each of them by the greatest value among all values reached. According to these results, one can observe that HS is very competitive.

4 Conclusion

In this study, a hybrid method was designed for efficiently tackling the bi-objective obnoxious p-median problem. The designed method starts with an archive set provided by tailoring an efficient greedy procedure. Next, intensification and diversification strategies were applied for enriching the diversity of the current Pareto front. Furthermore, an iterative ϵ-constraint procedure is applied for highlighting the final set of points, where both objective functions are alternatively considered. The behavior of the proposed method was experimentally analyzed, where both qualitative and quantitative studies were considered. Encouraging results have been obtained.

References

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