A Population-Based Algorithm for the Bi-Objective Quadratic Multiple Knapsack Problem

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1 Introduction

The knapsack problem arises in several real-world applications, like cutting and packing, scheduling, transportations, etc. Herein, we focus on approximately solving the Bi-Objective Quadratic Multiple Knapsack Problem (BO-QMKP), where it is tackled with a hybrid population-based method. Such a problem can be viewed as a combination of two well-known NP-hard combinatorial optimization problems with two different objective functions : the Quadratic Knapsack Problem−QKP and, the Multiple Knapsack Problem−MKP. The proposed method tries to combine several operators incorporated into the non-sorting genetic algorithm.

2 The problem

An instance of BO-QMKP is characterized by a set M of m knapsacks of fixed capacity each, i.e., $c = (c_1, \ldots, c_m)$, and a set N of n items. Each item i, $\forall\ i \in N$, is characterized by a profit p_i and a weight w_i and each pair of distinct items (i, j) belonging to $N \times N$ $(i \neq j)$ has an augmented profit p_{ij} if both items belong to the same knapsack $k,\ k \in M$. The goal of the problem is to assign each item to at most one knapsack such that the total weight of the items in each knapsack $k,\ k \in M$, does not exceed its capacity c_k and both (i) the total profit of all the items included into the knapsacks and (ii) the makespan related to the knapsack with the lowest gain, are maximized. Let $\boldsymbol{x_{ik}}$ be the decision variable set equal to 1 if the item $i,\ i \in I$, is assigned to the knapsack $k,\ k \in K$, 0 otherwise. Then, the formal description of BO-QMKP (noted $\mathrm{P}_{\mathrm{BO-QMKP}}$) is given as follows :

$$z_1(x) = \max \quad \sum_{i\in N}\sum_{k\in M} \boldsymbol{p_i x_{ik}} + \sum_{i\in N}\sum_{\substack{j\in N\\ i<j}}\sum_{k\in M} \boldsymbol{p_{ij} x_{ik} x_{jk}}$$

$$z_2(x) = \max \quad \min_{k\in M} \sum_{i\in N} \boldsymbol{p_i x_{ik}} + \sum_{i\in N}\sum_{\substack{j\in N\\ i<j}} \boldsymbol{p_{ij} x_{ik} x_{jk}} \tag{1}$$

$$\text{s.t.} \quad \sum_{i\in N} \boldsymbol{w_i x_{ik}} \leq \boldsymbol{c_k}, \forall k \in M, \tag{2}$$

$$\sum_{k\in M} \boldsymbol{x_{ik}} \leq 1, \forall i \in N, \tag{3}$$

$$\boldsymbol{x_{ik}} \in \{0,1\}, \forall i \in N,\ \forall k \in M, \tag{4}$$

3 A Hybrid Population Based Method For Solving The Bi-Objective Quadratic Multiple Knapsack Problem

The algorithm begins by building two reference solutions provided by using the adaptation of the branch and solve approach [1], and by adapting a special ε-constraint heuristic [2]. Further, a starting population of solutions $\mathcal{P}_0$ is created by calling a spacial "guided procedure" [1]. At each iteration, a tournament selection is applied in order to expand the search space. For each generation, namely t, a new population $\mathcal{Q}_t$ is created by applying the *fusion operator* and the so-called mutation operator to the current population $\mathcal{P}_t$. Then, the intensification strategy, which is based upon a descent method is called for expanding each generation.

	EPR		BS		ε-CSBH		HPBA (This work)	
#Inst	z_1	z_2	z_1	z_2	z_1	z_2	z_1	z_2
I300_75_3_1	589739	75543	589798	75543	589435	75751	589798	75504
I300_75_3_2	641610	59565	641650	59565	641610	59565	641610	59565
I300_75_3_3	598124	72391	598124	72391	598124	72391	598124	72391
I300_75_3_4	581227	72605	581227	72605	581031	73183	581031	73183
I300_75_3_5	612383	70948	612383	70948	611931	73106	612139	71200
I300_75_5_1	405191	30259	405191	30259	405191	30259	405191	30259
I300_75_5_2	445655	25108	445655	25108	445799	25138	**446129***	**25469***
I300_75_5_3	406800	30208	406800	30208	406800	30208	406800	30208
I300_75_5_4	396021	27448	396021	27448	396021	27448	396021	27448
I300_75_5_5	415804	30667	415804	30667	415804	30667	415804	30667
I300_75_10_1	248136	8205	248136	8205	248136	8205	248136	8205
I300_75_10_2	268003	6185	268007	5999	268102	6185	**268107***	**6206***
I300_75_10_3	239875	9993	239947	9957	240016	9957	**240070***	**9993***
I300_75_10_4	231812	8646	231858	8581	231858	8581	**231923***	**8646***
I300_75_10_5	249668	7994	249704	7832	249786	7625	249704	7832

TAB. 1 – Behavior of HPBA versus EPR, BS and ε-CSBH on a large set of Benchmark instances.

As shown in Table 1, the Hybrid Population Based Algorithm (HPBA) performs better than all existing methods of the literature. It is able to reach 4 new points (Pareto solutions) with both objective functions (z_1, z_2) : instances `I300-25_10_3`, `I300-75_5_2`, `I300-75_10_2` and `I300-75_10_4`.

4 Conclusion

We investigated the use of a hybrid multi-objective evolutionary algorithm for solving the bi-objective quadratic multiple knapsack problem. The proposed algorithm is based upon the so-called non-dominated sorting operator, where both the general profit and the min-max criterion on the second objective function are optimized. A starting population of solutions were provided by combining two starting solutions : a first solution provided by a special ε-constraint heuristic and, (ii) a second solution achieved by an adaptive local-branching-based heuristic. In order to highlight the quality of the solutions reached, a drop/rebuild strategy was incorporated into the search process. Computational results showed that the proposed method remains competitive (in term of quality of the achieved bounds), especially when comparing its provided bounds to those reached by the more recent methods published in the literature.

Références

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