# Single-leader multi-follower games for the regulation of two-sided Mobility-as-a-Service markets

Haoning Xi<sup>1</sup>, Didier Aussel<sup>2</sup>, Wei Liu<sup>3</sup>, S. Travis Waller<sup>1</sup>, David Rey<sup>4</sup>

<sup>1</sup> School of Civil and Environmental Engineering, UNSW Sydney, NSW, 2052, Australia {haoning.xi,s.waller}@unsw.edu.au

<sup>2</sup> PROMES UPR CNRS 8521, Université de Perpignan Via Domitia, Tecnosud, Perpignan, France aussel@univ-perp.fr

 $^{3}\,$  Department of Aeronautical and Aviation Engineering, The Hong Kong Polytechnic University,

Hong Kong

wei.w.liu@polyu.edu.hk

<sup>4</sup> SKEMA Business School, Université Côte d'Azur, Sophia Antipolis Campus, France david.rey@skema.edu

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### 1 Introduction

The transportation industry is undergoing a massive revolution from infrastructure-focused towards service-focused business models brought by the "Mobility-as-a-Service (MaaS)" concept. MaaS is a user-centric paradigm where customized services and mode-agnostic mobility resources are priced in a unified framework [2]. We consider a MaaS system, which admits a natural two-sided market representation wherein a MaaS regulator aims to promote the participation of both travelers and transportation service providers (TSPs) [1]. The operation of a two-sided MaaS market is challenging due to the interactions among different stakeholders, i.e. the regulator, travelers and TSPs which may have divergent and conflicting objectives. To capture the interactions across stakeholders of a two-sided MaaS market, we propose a singleleader multi-follower game (SLMFG) where the MaaS regulator is the leader, and travelers and TSPs are two groups of followers. In this SLMFG, the MaaS regulator aims to maximize its profits by allocating mobility resources supplied by TSPs to travelers. Each traveler aims to minimize her travel costs by deciding the participation level defined as the proportion of mobility demand fulfilled through the MaaS platform in comparison with reserve options. Each TSP aims to maximize its profits by deciding the participation level defined as the proportion of mobility resources supplied to the MaaS platform in comparison with its reserve options. We consider an auction framework where travelers and TSPs bid through the MaaS platform and propose several SLMFGs. The resulting mathematical optimization problems can be represented as mixed-integer bilevel programming problems. We propose exact solution methods based on customized branch and bound algorithms that exploit the structure of these problems.

### 2 Single leader multi follower game formulation

Let  $\mathcal{I}$  denote the set of travelers,  $\mathcal{M}$  denote the set of travel modes and  $\mathcal{N}_m$  denotes the set of TSPs of mode  $m, \forall m \in \mathcal{M}$ . We denote p and q the non-negative real variable representing the unit threshold price for travelers and TSPs, respectively. The MaaS regulator accepts or rejects participants by comparing their bidding prices with the internal threshold unit price. We denote  $[u_i]_{i\in\mathcal{I}}$  and  $[w_{mn}]_{m\in\mathcal{M},n\in\mathcal{N}_m}$  the binary variables indicating whether the MaaS regulator





accepts travelers and TSP, respectively. The allocation of mobility resources supplied by TSPs to travelers is coordinated via real variables  $l_i^m \geq 0$ , which represent the service time of travel mode  $m \in \mathcal{M}$  in the *MaaS bundle* of traveler  $i \in \mathcal{I}$ . Cross network effects in the two-sided market are captured by the real variable  $\Delta$  which represents the supply-demand gap based on the decisions of followers. The decision space of the leader is thus the tuple  $(p, q, u, w, l, \Delta)$ where bold-face symbols represent vectors of variables of appropriate dimensions. The decision variables of followers are divided into two groups of variables  $\boldsymbol{x} = [x_i]_{i \in \mathcal{I}}$  for travelers and  $\boldsymbol{y} = [y_{mn}]_{m \in \mathcal{M}, n \in \mathcal{N}_m}$  for TSPs. For each traveler  $i \in \mathcal{I}, x_i \in [0, 1]$  is a real variable representing the proportion of mobility demand that i decides to use the MaaS platform in comparison with her other options, e.g., private vehicles and public transit. Analogously, for each TSP  $mn, m \in \mathcal{M}, n \in \mathcal{N}_m, y_{mn} \in [0,1]$  is a real variable representing the proportion of mobility resources that TSP mn decides to supply the MaaS platform as opposed to its reserve options. Let  $F(\cdot)$ ,  $G_i(\cdot)$  and  $H_{mn}(\cdot)$  be the objective functions of the regulator, traveler  $i \in \mathcal{I}$  and TSP  $m \in \mathcal{M}, n \in \mathcal{N}_m$ , respectively. The SLMFG base model for two-sided MaaS market is illustrated in Figure 1. We derive this SLMFG framework into several mixed-integer bilevel programming formulations that vary based on the modeling of cross-network effects, i.e the level of interaction across both groups of followers in the two-sided market.

## 3 Solution approach and findings

To solve the proposed mixed-integer bilevel programming problems, we provide constraint qualifications for MPEC reformulations of the SLMFGs and prove the equivalence between the MPEC reformulations and their original problems, which lays a basis to develop solution algorithms based on the single-level reformulation of the SLMFGs. We propose an exact branch and bound algorithm based on the strong duality reformulation of the SLMFGs which branches on accept/reject binary variables, and explore three rules to select branching variables. Our computational results demonstrate the performance of the proposed SD-based algorithm relative to a benchmark branch and bound algorithm based : the computational speed of the proposed approach is over  $10 \sim 100$  times faster than that of the benchmark when the scale of the instances is large.

#### Références

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