Bi-objective edge-based capacitated single-server queuing-location problem

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1 Introduction

Location problems aim to determine the best locations for establishing facilities, such that the designated objective is reached. Although the main objective varies with the type of studied location problem, it could be summarized as minimizing: 1) the total traveling times in median problems, 2) the total number of uncovered demands in covering problems, and 3) the worst distance or service times in center problems. Queuing-location refers to congested location problems in which customers need to wait in a line until being served.

Although in real-world cases customers are distributed along the streets that could be interpreted as network edges, the majority of location problems are node-based, considering the demands located at network nodes. Inspired by [2] considering edge-based demands, this work reflects a more realistic image of the problem by studying a capacitated bi-objective version in which customers are uniformly distributed along the network edges. The problem is discrete with a pre-known set of candidate locations. The assumptions of this study could be summarized as follows. For each network edge, the demand generation follows a Poisson distribution. As it is illustrated in Fig. 1 (A), network edges are decomposed into two segments, each assigned to its closest facility. The decomposing point has the same distance to its corresponding closest facilities. It is worth mentioning that if the closest facilities are identical, the edge is entirely assigned to that facility, without being decomposed. The movement of customers to arrive at each facility could be interpreted as an $M/G/\infty$ queuing system that yields Poisson outputs entering each facility. To receive the service, arriving demands need to wait in a line after entering each facility. A finite capacity (K) has been considered for the queue limiting the number of entering customers, such that no more than K customers can be in line. If the waiting capacity is full, arriving customers leave the facility without receiving the service. These customers are called lost demands. Each facility contains one server for which the service time follows an Exponential distribution. Therefore, the studied congested system at each facility would be within an $M/M/1/K$ queuing framework [3]. Such assumptions holds in most competitive facilities such as ATMs, medical diagnosis facilities, and telecommunication service facilities.

It is assumed that the cost of establishing facilities at each candidate location is known. As a combination of median and covering location problems, this work is studied in a bi-objective framework. The first objective minimizes the total cost of establishing facilities as the summation of establishing costs of all opened facilities. The second objective minimizes the summation of the total traveling cost, the total expected waiting cost of customers, and the total cost of demands lost due to the waiting room's capacity restriction. The main constraints could be summarized as follows: Each customer is assigned only to the closest opened facility; the expected waiting time at each facility should not exceed a pre-defined threshold. Since location problems are known to be NP-hard on a general graph and due to the extreme non-

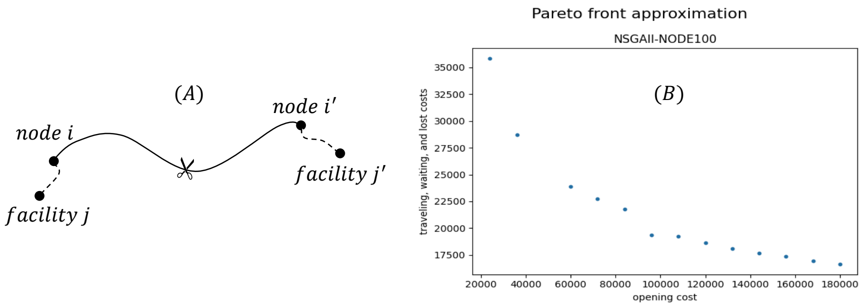

FIG. 1: (A) Arc decomposition (B) The obtained non-dominated solutions

linearity of related formulations, the Non-dominated Sorting Genetic Algorithm–II (NSGA-II) is applied as the solution method [1].

2 Methodology

To implement the NSGA-II, each solution is represented as a binary vector with a length of candidate sites, such that "ones" indicate the candidate sites selected for establishing the facilities. Each network edge is decomposed based on the distance of its corresponding endpoints to the opened sites, such that each segment of the network edges is assigned to its closest opened facility. Based on the assignments of network edges, the total traveling distance and number of entering demands to each facility could be calculated. The expected waiting time and the number of lost demands could be calculated using the number of entering demands. In the case of violation of waiting time threshold constraint, a penalty function is added to the objective value of infeasible solutions. This study applies a two-point crossover operator for mating selected parents. The mutation operator selects a random number of genes along the chromosome and changes the status of the corresponding candidate locations. The penalty function is also applied to the generated infeasible offspring.

3 Conclusions and perspectives

In this work, a single-server capacitated discrete bi-objective location problem subject to an M/M/1/K queuing framework is studied. The first objective is to minimize the total establishing costs, while the second objective aims to minimize the total traveling cost, the total expected waiting cost, and the total lost demands' cost. The effectiveness of applied NSGA-II algorithm in solving the problem through several generated test cases is analyzed using unary hypervolume (Fig. 1 (B) shows the obtained non-dominated solutions for one of the test cases). It might be interesting to develop other algorithms to evaluate their performance.

References

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