Branch-Cut-and-Price algorithm for an Operational Storage Location Assignment Problem

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1 Introduction

In internal warehousing logistics, the placement of different items, i.e. SKU (Stock Keeping Units), is one of the most impactful decisions on the performance of warehouse activities. The Storage Location Assignment Problem (SLAP) aims at finding an efficient assignment of the SKU to the storage locations. However, the storage decisions must be strongly connected to the routing of the order pickers, to assess the performances of an allocation. In this talk, we focus on the forward area (also called fast pick area), where the storage decisions change dynamically, with a very versatile demand. It is therefore interesting in this context to consider the problem with full information on the demand over a time horizon, with a *pick-by-order* structure, where all the items of one order are picked altogether.

In the related literature, the Storage Location Assignment Problem (SLAP) has been extensively studied, almost only with class-based storage and frequency information on SKU. Most of the state-of-the-art models have been designed to be easy to implement and to tackle industrial-scale instances. Various solutions methods have been proposed in the form of policy-based heuristics [3]. However, the *pick-by-order* feature, where all SKU of one order are picked altogether in one route, is often not considered with exact distances in mathematical programming, because of the class-based storage organization. Furthermore, recent work has been made on the integration of storage and routing decisions, but mostly with simulation methods [2]. Few attempts have been made at tackling the problem with full information on demand and order structure [3], and even less with exact methods, the article [1] being the only example to the best of our knowledge. The aforementioned paper focuses on complexity analysis, and solves the problem with a dynamic programming approach. The paper [4] studies a similar problem, and developed a heuristic algorithm to tackle industrial-scale instances. It accounts for the full information about the by-order structure of the demand, applied to high-level warehouses. Its numerical results are competitive compared to the state-of-the-art class-based methods.

Having a highly-constrained and combinatorial structure, the SLAP problem presents a strong academic interest to get a more thorough comprehension of the interactions between the layout, storage and routing decisions. This version of the SLAP problem was proven NP-hard in the strong sense, even in very simple cases [1], therefore it is particularly challenging for exact methods. It also has a practical interest to model forward areas, where the storage assignment changes dynamically in a very versatile context. In forward areas, the storage decisions are thus posed at the operational level, and the picking policy gets a bigger impact.

2 Extended formulation

In this talk, we introduce a novel formulation for the storage location assignment problem, accounting for the *by-order* structure of the demand. The proposed formulation presents a highly-structured constraint matrix, which is prone to decomposition methods. The Dantzig-Wolf reformulation is applied to the SLAP, leading to an integer linear program, which presents the advantages of being independent from both: **(i)** the warehouse layout, and **(ii)** the chosen routing policy for order pickers. The aforementioned two aspects are convexified in the subproblems.

3 Solution Method

Since the exponential number of decision variables in this formulation does not permit a tractable enumeration, the relaxation is solved by column generation, with a decomposition of the pricing problems. The relaxation of the integrality constraints leads to the master problem, then restricted to a subset of columns. Additional columns are integrated dynamically at each iteration, by looking for routes of negative reduced cost. The highly structured constraint matrix of the problem leads to a natural decomposition for route generation. Indeed, one pricing problem needs to be solved for each order, and all problems are independent. The warehouse layout and routing policy for order pickers appear explicitly in these problems, so the formulation of the pricing changes depending on the characteristics of the problem at hand. The subproblems are modeled with an adapted resource-constrained shortest path problem, solved by dynamic programming.

Several families of valid inequalities have been identified to strengthen the dual bound provided by the column generation. Polynomial-size families are added directly to the problem at the root node. Valid inequality families, whose cardinality does not allow an exhaustive enumeration, are generated dynamically by an efficient separation procedure.

4 Results & Conclusions

Preliminary results are encouraging. The average gap between the dual bound at root node and the best known solution over a set of test instances passes from 85.8% with an MIP formulation to 23.1% with the extended formulation. The addition of valid inequalities improves further this result, with an average gap of 10.6%. The Branch-Cut-and-Price solution method presented in this talk clearly outperforms commercial solvers in the preliminary experiments. Further work is ongoing to speed up the procedure.

References

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