# Sharing the Cost of a Gas Distribution Network.

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## 1 Introduction

Gas distribution is achieved by mean of a network that draws gas from a source and channels it to the consumer. Such network is managed by an operator, which is responsible for delivering natural gas while ensuring the safety and maintenance of the network. In order to convey out its task properly, a network operator is confronted with various **costs**, some of which are not directly assignable to a given consumer.

Usually, a network operator recovers the costs of operating the network via a distribution rate. The competitiveness of the network operator depends significantly on the relevance of this rate. In particular, a distribution rate is relevant if it meets the **ethical principles** pursued by the network operator.

To assist decision makers in selecting a relevant distribution rate, **cost sharing rules** are introduced. Meanwhile, three principles are discussed : the **independence of higher demands** principles, the **connection** principle and the **equity** principle. This allows us to adopt a normative approach in order to determine relevant cost sharing rules, and to conduct an **axiomatic study**. To that end, a cost sharing problem adapted to the gas distribution setting is defined.

## 2 The Model

Fix a finite set $N = \{1, \ldots, n\}$ of **consumers**, directly or indirectly, connected to a source in gas by mean of a **fixed network**, which is represented by a directed tree graph $P$. The nodes of the graph represent the consumers plus the source, while the arcs of the graph represent the pipelines of the network. An integer in $N$ refers to both a consumer and the pipeline having this consumer at its tail. For instance, in FIG. 1, the red pipeline is called "pipeline 2".

Each consumer $i \in N$ has an **effective demand** $q_i \in \mathbb{N}$. This effective demand corresponds to the highest daily volume that this consumer expects to achieve in a year. For instance, the effective demand of a regular household is determined by its consumption during winter. The effective demand is communicated to the operator in advance in order to design the network accordingly. The profile of all effective demands is given by $q = (q_1, \ldots, q_n)$. Without loss of generality, assume that $q_n \geq q_i$, for each $i \in N$. Denote by $Q(j) = \{i \in N : q_i \geq j\}$ the set of consumers with an effective demand of at least $j$.

Each consumer $i \in N$ is endowed with a discrete set $\{0, 1, \ldots, q_i\}$ that describes its effective demand $q_i$ and each of the demands smaller than $q_i$. This set can be interpreted as the set of all **demands available** to $i$, since $i$ is not required to demand $q_i$ all year round.

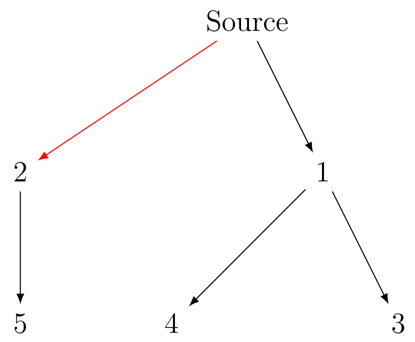


FIG. 1 – Distribution network $P$

A cost function $C : N \times \{1, \ldots, q_n\} \to \mathbb{R}_+$ that computes the cost of any pipeline is introduced. Formally, for each $i \in N$ and $j \in \{1, \ldots, q_n\}$, $C(i, j)$ represents the cost of operating the pipeline $i$ when it is designed to meet a demand of $j$. The convention $C(i, 0) = 0$, for each $i \in N$, is used. Each map $C(i, .) : \{1, \ldots, q_n\} \to \mathbb{R}_+$, $i \in N$, is non-decreasing : the larger the demand, the wider the pipeline, which leads to greater costs. On the other hand, for each $j \in \{1, \ldots, q_n\}$ and any two $i, i' \in N$, we do not necessarily have $C(i, j) = C(i', j)$ since two pipelines are not always identical. Equivalently, a cost function can be expressed as a **matrix of incremental costs**. Denote $A^C_{ij}$ the **incremental cost** of pipeline $i$ for a demand $j$. The incremental cost $A^C_{ij}$ represents the increase in cost of pipeline $i$ when it is upgraded to meet a demand of $j$ instead of $j - 1$, formally

$$\forall i \in N, \forall j \leq q_n, \quad A^C_{ij} = C(i, j) - C(i, j - 1). \tag{1}$$

Since the network operator must satisfy any effective demand at any time, each pipeline should be wide enough to meet the highest effective demand of the consumers located at the tail and downstream of this pipeline (highest downstream demand for short). The **total cost** of operating the network is computed as the sum of the costs of all the pipelines designed to meet their highest downstream demand for short, which is given by

$$\text{Total cost} = \sum_{i \in N} \sum_{j \in \{1, \ldots, \overline{q}_i\}} A^C_{ij}. \tag{2}$$

The problem is then to determine a relevant way to recover to the total cost (2). This problem is called the **gas distribution problem** and is denoted by $(q, A^C)$. The class of of all gas distribution problems is denoted by G. To solve this problem, cost sharing rules are introduced. A cost sharing rule refers to a map

$$f : (q, A^C) \to f(q, A^C) \in \mathbb{R}^{q_1 + \ldots + q_n}_+. \tag{3}$$

A cost sharing rule specifies a payoff $f_{ij}(q, A^C)$ to each demand $j \in \{1, \ldots, q_i\}$ of each consumer $i$. A cost sharing rule provides more information than a solution that would have split (2) in $n$ shares i.e. among the consumers. This additional information will prove useful for the rest of our study. A cost sharing rule satisfies the budget balanced condition, meaning that it recovers the total cost of operating the network.

A normative approach to determine relevant cost sharing rules is adopted. To that end, three principles for cost sharing rules are discussed.

- The first principle is the **independence of higher demands** principle. It indicates that a consumer should not have to pay for demands higher than its own. In other words, any costs incurred by demands higher than its own effective demand should not be charged to that consumer. This principle makes it possible to avoid situations where the presence of a consumer with a high demand implies additional and unjustified costs to consumers with lower demands. For example, it seems unjustified to increase the bill of a household whenever a factory with a large consumption moves in next door.

– The second principle is called the **connection** principle. Consider a network in which all consumers, except one, have a null effective demand. To supply this consumer, all the pipelines connecting this consumer to the source must be involved. In other words, this consumer depends on a number of pipelines in addition to the one to which it is the tail. It should be reasonable to only charge this consumer for the costs of operating these pipelines. In a more general way, consumers should only pay for the portion of the network they use. This idea is the connection principle.
– The last principle is called the **equity** principle. This principle simply states that two consumers with the same demands should be charged with the same amount regardless of their geographical location. For instance, there should be no difference in terms of distribution rate applied in rural areas compared to urban areas, although the underlying costs are different. Obviously, the equity principle is an egalitarian principle. In fact, network operators in France strive to offer distribution rates that respect this principle as much as possible.

# 3 The results

## 3.1 The Connection rule

Applying the independence of higher demands principle and the connection principle, we derive the Connection rule. This rule ensures that each consumer pays an appropriate share of the portion of the network that connects it to the source. Take any $i \in N$ and $j \leq q_i$. For each pipeline $k$ located upstream of $i$, consumer $i$ has to pay a fair share of $A^C_{kj}$ for its demand $j$.

**Definition 1** The Connection rule $\Psi$ is defined, for each $(q, A^C) \in \mathrm{G}$, as

$$\forall i \in N, \forall j \leq q_i, \quad \Psi_{ij}(q, A^C) = \sum_{k \in \hat{P}^{-1}(i) \cup i} \frac{A^C_{kj}}{(\hat{P}(k) \cup k) \cap Q(j)}. \tag{4}$$

To emphasize our normative approach, an axiomatic characterization of the Connection rule is provided. Let $f$ be a rule on G. The first axiom, Linearity, is a classical axiom in the cost sharing literature.

**Linearity** For each $(q, A^C), (q, A^{C'}) \in \mathrm{G}$ and $\beta \in \mathbb{R}$,

$$\forall i \in N, \forall j \leq q_i, \quad f_{ij}(q, A^C + \beta A^{C'}) = f_{ij}(q, A^C) + \beta f_{ij}(q, A^{C'}). \tag{5}$$

The independence of higher demands principle is translated into an axiom. This axiom compares two situations. The first situation is a gas distribution problem. The second situation is another gas distribution problem similar to the first one, but in which it is no longer possible for consumers to demand more than a certain quantity, let us say $l$. This axiom requires that the amount charged to any consumer for any demand below $l$ remains the same in both situations.

**Independence of Higher Demands** For each $(q, A^C) \in \mathrm{G}$ and each $l \in \{0, \ldots, q_n\}$,

$$\forall i \in N, \forall j \leq q_i, \quad f_{ij}(q, A^C) = f_{ij}((l \wedge q_k)_{k \in N}, A^C). \tag{6}$$

A pipeline is **irrelevant** to a consumer if it is not a pipeline that helps to connect the consumer to the source. It seems reasonable to not charge a consumer for the costs generated by pipelines irrelevant to it. In other words, a consumer can only be charged for the costs generated by the pipelines that connect it to the source. This idea is formalized in the next axiom. This axiom compares two gas distribution problems. The two problems may differ in their incremental costs matrix. However, there exists a path from the source to a certain consumer $i \in N$, in which the incremental costs generated, for a certain demand $j$, by all the pipelines constituting this path are identical in both problems. The consumers located on this path are charged the same amount in both problems for their demand $j$.

**Independence of Irrelevant Costs** For each $(q, A^C), (q, A^{C'}) \in \mathrm{G}$, $j \in \{0, \ldots, q_n\}$ and $i \in N$, such that $A^C_{hj} = A^{C'}_{hj}$ for each $h \in \hat{P}^{-1}(i) \cup i$,

$$\forall h \in (\hat{P}^{-1}(i) \cup i) \cap Q(j), \quad f_{hj}(q, A^C) = f_{hj}(q, A^{C'}). \tag{7}$$

Each downstream consumer should be treated equally regarding a given upstream cost. Consider a gas distribution problem in which only one pipeline generates non null incremental costs for a given demand, let us say $j$. All consumers located downstream of the cost generating pipeline should be charged the same amount for their demand $j$.

**Downstream Symmetry** For each $(q, A^C) \in \mathrm{G}$, $i \in N$ and $j \in \{0, \ldots, q_n\}$, such that $A^C_{kj} = 0$ for all $k \neq i$,

$$\forall h, h' \in \hat{P}(i) \cup i, \quad f_{hj}(q, A^C) = f_{h'j}(q, A^C). \tag{8}$$

**Theorem 1** *A rule $f$ on* G *satisfies Linearity, Independence of Higher Demands, Independence of Irrelevant Costs and Downstream Symmetry if and only if it is the Connection rule.*

## 3.2 The Equity rule

Applying the independence of higher demands principle and the equity principle, we derive the Equity rule. Take any $i \in N$ and $j \leq q_i$. The equity rule divides each $A^C_{ij}$ equally among the demand $j$ each consumer $k$ verifying $q_k \geq j$.

**Definition 2** The Equity rule $\Upsilon$ is defined, for each $(q, A^C) \in \mathrm{G}$, as

$$\forall i \in N, \forall j \leq q_i, \quad \Upsilon_{ij}(q, A^C) = \frac{1}{|Q(j)|} \sum_{k \in \hat{P}^{-1}(Q(j)) \cup Q(j)} A^C_{kj}. \tag{9}$$

Assume that one or several pipelines generate additional costs due to exogenous reasons (incident, natural disaster, etc), which leads to an increase of the incremental costs. Such additional costs should not increase the inequalities between the cost shares of the consumers. Formally, consider any demand $j$. Compare the difference in cost share between two consumers in $Q(j)$ before and after the costs increase. In particular, compare the difference between the highest cost share charged to a consumer for its demand $j$ with the lowest cost share charged to a consumer for its demand $j$. If the incremental costs increase, then this difference should not increase to aggravate inequalities. This idea is in line with the equity principle.

**Non-Increasing Inequalities** For each $(q, A^C), (q, A^{C'}) \in \mathrm{G}$ such that $A^{C'}_{ij} \geq A^C_{ij}$, for each $i \in N$ and $j \leq q_n$,

$$\forall j \in \{1, \ldots, q_n\}, \quad \max_{i \in Q(j)} f_{ij}(q, A^{C'}) - \min_{i \in Q(j)} f_{ij}(q, A^{C'}) \leq \max_{i \in Q(j)} f_{ij}(q, A^C) - \min_{i \in Q(j)} f_{ij}(q, A^C). \tag{10}$$

**Theorem 2** *A rule $f$ on* G *satisfies Independence of Higher Demands and Non-Increasing Inequalities if and only if it is the Equity rule.*

### 3.3 The Mixed rules

Observe that a cost sharing rule can hardly satisfy both the connection principle and the equity principle on the full class of gas distribution problems. Indeed, the connection principle states that consumers should only pay for the portion of the network they use. Therefore, two consumers with the same demands can be charged with different amount depending on their position on the network, which contradicts the equity principle.

To compromise between these two principles, a family of cost sharing rules is proposed. Each cost sharing rule in this family compromises between the connection principle and the equity principle by means of convex combinations.

**Definition 3** Let $\alpha = \{\alpha^j\}_{1\leq j\leq K}$ be a parameter system such that $\alpha^j \in [0,1]$, for each $1 \leq j \leq K$. The $\alpha$-Mixed rule $\mu^\alpha$ is defined, for each $(q, A^C) \in \mathrm{G}$, as

$$\forall i \in N, \forall j \leq q_i, \quad \mu^\alpha_{ij}(q, A^C) = \alpha^j \Psi_{ij}(q, A^C) + (1 - \alpha^j)\Upsilon_{ij}(q, A^C). \tag{11}$$

A $\alpha$-Mixed rule operates convex combinations between the Connection rule and the Equity rule. For each demand $j \in \{0, \ldots, q_n\}$, a consumer $i \in Q(j)$ is charged a cost share lying between $\Psi_{ij}(q, A^C)$ and $\Upsilon_{ij}(q, A^C)$. If $\alpha^j$ is closer to 1, then this cost share is closer to $\Psi_{ij}(q, A^C)$, and is more in line with the connection principle. On the other hand, if $\alpha^j$ is closer to 0, then this cost share is closer to $\Upsilon_{ij}(q, A^C)$, and is more in line with the equity principle.

The parameter system $\alpha$ determines which kind of compromises occurs at each level of demand. For instance, it is possible to promote the equity principle for small demands and the connection principle for large demands e.g $\alpha = (1, 0.9, \ldots, 0.1, 0)$. This option of making several compromises is only possible because we consider rules that assign a payoff to each demand of each player.

To characterize the Mixed rules, we introduce two new axioms. Let $f$ be a rule on G. The first axiom addresses irrelevant costs in the sense of the Independence of Irrelevant Costs axiom. It indicates that two consumers should be equally impacted by irrelevant costs, without specifying the extent of this impact.

**Equal Impact of Irrelevant Costs** For each $(q, A^C), (q, A^{C'}) \in \mathrm{G}$, $j \in \{0, \ldots, q_n\}$ and $i \in N$, such that $A^C_{hj} = A^{C'}_{hj}$ for each $h \in \hat{P}^{-1}(i) \cup i$,

$$\forall h, h' \in (\hat{P}^{-1}(i) \cup i) \cap Q(j), \quad f_{hj}(q, A^C) - f_{hj}(q, A^{C'}) = f_{h'j}(q, A^C) - f_{h'j}(q, A^{C'}). \tag{12}$$

The second axiom compares the cost share charged the consumers on the basis of their geographical localization. Consider a gas distribution problem in which only one pipeline generates non-null incremental costs for a given demand, let us say $j$. The consumer $i$, located on the tail of the cost generating pipeline, should be charged more than any other consumer for its demand $j$. Meanwhile, the axiom does not compare the cost share of two consumers different from $i$.

**Fairness** For each $(q, A^C) \in \mathrm{G}$, $i \in N$ and $j \in \{0, \ldots, q_n\}$, such that $A^C_{hj} = 0$ for all $h \neq i$,

$$\forall h \neq i, \quad f_{ij}(q, A^C) \geq f_{hj}(q, A^C). \tag{13}$$

**Theorem 3** *A rule on* G *satisfies Linearity, Independence of Higher Demands, Equal Impact of Irrelevant Costs, Fairness and Downstream symmetry if and only if it is a Mixed rule.*

# 4 Additional remarks

An appropriate game theoretic tool for modeling gas distribution problems are multi-choice (cooperative) games. Multi-choice games, introduced by [2], are a natural extension of TU-games in which each player is endowed with a certain number of activity levels. A (multi-choice) coalition is a profile describing each player's activity level within this coalition. The worth of each coalition is measured by a characteristic function. Given a gas distribution problem, a special multi-choice game associated with this problem is derived. This game is called the gas distribution game. The player set represents the set of consumers, and the activity levels represent the demands of the consumers. The worth of a coalition corresponds to the cost of the network designed in order to meet the demands of this coalition. The Connection rule applied to a gas distribution problem corresponds to the multi-choice Shapley value, introduced by [3], of the corresponding gas distribution game. Similarly, the Equity rule corresponds to the multi-choice Equal division value and the Mixed rules to the multi-choice Egalitarian Shapley values. Both values are introduced by [3]. Moreover, for each gas distribution problem, the multi-choice Shapley value of the corresponding gas distribution game is in the core (as defined by [1]) of this gas distribution game.

# 5 Conclusion

Gas distribution problems are defined and three rules for these problems are determined on the basis of ethical principles. Applying the Connection principle and the Independence of higher demands principle, the Connection rule is derived. Applying the equity principle and the Independence of higher demands principle, the Equity rule is derived. To make a trade-off between the Connection principle and the equity principle, the Mixed rules are derived. An axiomatic characterization for each one of these rules is proposed. Finally, we point out that our cost sharing rules relate to solution concepts from multi-choice games.

# Références


[1] M. Grabisch and L. Xie. A new approach to the core and Weber set of multichoice games. *Mathematical Methods of Operations Research*, 66(3) :491–512, 2007.

[2] C.-R. Hsiao and T. Raghavan. Monotonicity and dummy free property for multi-choice cooperative games. *International Journal of Game Theory*, 21(3) :301–312, 1992.

[3] D. Lowing and K. Techer. Marginalism, egalitarism and efficiency in multi-choice games. *Working Paper*, 2021.