Adiabatic quantum optimization for two classical problems

Gérard Fleury and Philippe Lacomme

Université Clermont Auvergne, LIMOS UMR CNRS 6158, 63178 Aubière Cedex, France

gerard.fleury@isima.fr, placomme@isima.fr

Keywords*: quantum computing, Adiabatic Optimization.*

1 Introduction

Currently, there is much interest in using adiabatic quantum optimization (AQO) and quantum simulated annealing to solve both NP-complete or NP-hard problems. These approaches received a lot of attention, because if a quantum Hamiltonian H_p can be defined for one problem, the ground state encodes the solution(s) of the problem. Quantum computing is a combination of quantum physic, computer science and mathematical theory that can be used now to create circuits which can be executed on simulators and/or real quantum computers provided for example, by IBM, DWave or Rigetti…

Quantum heuristics can significantly expand the list of applications for which quantum computers have a significant advantage in the same trend of researches that has crossed the classical operational research community from decades in numerous areas, including but not limited to scheduling and routing. Quantum approximate optimization algorithms take advantage of alternation between the cost function investigation which is modeled by H_p and the mixing Hamiltonian operator H_D. The quantum alternating operator ansatz takes into consideration a general parameterized family of unitary operators to efficiently modelize the Hamiltonian.

Lately Farhi et al [1] in 2002 introduced a promising Quantum Approximate Optimization Algorithm which formally defines a quantum alternating operator ansatz (QAOA) supporting a large and potentially useful set of states which create an alternative to the Adiabatic Optimization.

To find a solution, quantum approximate optimization algorithms allows to solve the Schrödinger equation

$$\frac{\partial.}{\partial t}|\psi(x,t)\rangle = -\frac{i}{\hbar}.H(t).|\psi(x,t)\rangle$$

If H is time independent the solution is $|\psi_t\rangle = e^{-\frac{i}{\hbar}t.H}.|\psi_0\rangle$ and is $|\psi_T\rangle = e^{-\frac{i}{\hbar}\int_o^T H(u).du}.|\psi_0\rangle$ in the general case.

At a first step, the system must be tuned in the ground state of one Hamiltonian H_D commonly referred to as the "driver Hamiltonian" that must be defined in such a way that it does not commute with the Hamiltonian modelling the problem. The adiabatic quantum optimization requires a Hamiltonian of the system is slowly modified from H_D to H_P using a interpolation based on one parameter $s(t)$ supposed very smoothly decreasing from 1 to 0, for instance (Fig. 1):

$$H(t) = s(t).H_D + [1 - s(t)].H_P$$

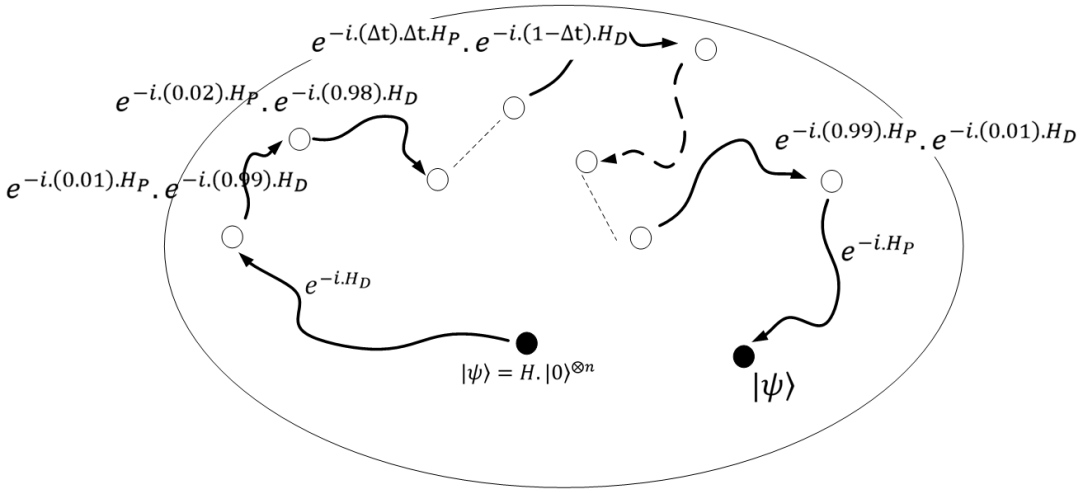

FIG. 1. Classical curve on one adiabatic process

2 Transportation problems: resolution

Several experiments are achieved, considering the Hamiltonian modelization for both the boolean satisfiability problem (SAT) and the graph partitioning in order to obtain valuable experimentations to comfort the theoretical consideration. The results shown us that the adiabatic quantum optimization permits to efficiently solve SAT problems (Fig. 2) and the graph partitioning one with the qiskit library using the IBM simulator.

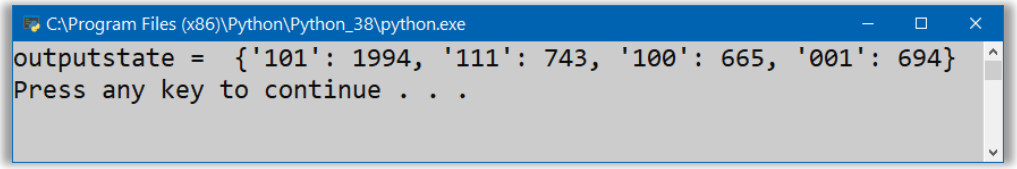

FIG. 2. Example of solution for the SAT problem

3 Concluding remarks

The adiabatic quantum optimization (AQO) defines a new resolution paradigm, strongly different to the widely used paradigms in operation research. But it requires a well-defined and very well-tuned Hamiltonian and a good skill at operators. The main difficulties for adiabatic resolution lies on the fact that such approaches are based on a combination of quantum physic, computer science and mathematical theory.

4 Reference

[1] E. Farhi, J. Goldstone and S. Gutmann. "Quantum Adiabatic Evolution Algorithms versus Simulated Annealing". Quantum Physics (quant-ph). arXiv:quant-ph/0201031. 2002.